CONTENTS

	Page	
1.0 SUMMARY	1	1/A9
2.0 INTRODUCTION	3	1/A11
3.0 SYMBOLS AND ABBREVIATIONS	4	1/A12
4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION	8	1/B2
4.1 The Basic Polynomial Equation	8	1/B2
4.2 Open-Loop and Closed-Loop Systems	10	1/B4
4.3 Transfer Function Formation	13	1/B7
4.4 Eigenvalue Calculation	15	1/B9
4.5 State Space Formulation	16	1/B10
4.6 Frequency Response Calculation	17	1/B11
4.7 Power Spectral Density Analysis	18	1/B12
4.8 Time Response Calculations	21	1/C1
4.8.1 Time Response by Method of Residues	21	1/C1
4.8.2 Time Response With Tau	23	1/C3
4.8.3 Time Response for Forcing Functions that Do Not Have Simple Laplace Transforms	23	1/C3
4.9 Unsteady Aerodynamic Options	24	1/C4
4.9.1 Frequency Response Analysis	25	1/C5
4.9.2 Flutter Analysis	27	1/C7
5.0 PROGRAM STRUCTURE AND DESCRIPTION	29	1/C9
5.1 QR Program Control and Eigenvalue Rooting	29	1/C9
5.2 Time Response Calculations	29	1/C9
5.3 Frequency Response and PSD Calculations	31	1/C12
5.4 Root Locus and V-g Plotting	31	1/C12
5.5 Unsteady Airforce Frequency Response and Flutter Calculations	31	1/C12
6.0 COMPUTER PROGRAM USAGE	32	1/C13
6.1 Control Cards	32	1/C13
6.2 Resource Estimates	33	1/C13
6.3 Card Input Data	34	1/D1
6.3.1 Case Initialization	35	1/D2
6.3.2 Matrix Element Operations	36	1/D3
6.3.3 Roots of Matrix and Root Locus	57	1/E10
6.3.4 Transfer Function Formation	69	1/F8
6.3.5 Time Response	73	1/F12
6.3.6 Frequency Response	79	1/G4
6.3.7 Unsteady Aerodynamics Options	84	1/G9
6.3.8 Cycling Control Cards and Labeling	94	2/A7
6.3.9 Input Card Summary	98	2/A11

CONTENTS (CONCLUDED)

Page

6.4 Examples of Required Card Sets for Some Typical Problems 107 2/C1
6.5 Output Data 109 2/C3
6.5.1 Printed Output Data 109 2/C3
6.5.2 File Output 109 2/C3
6.6 Restructions 115 2/C9
6.7 Diagnostics 115 2/C9
6.7.1 READTP Error Codes 118 2/C12
6.7.2 WRTETP Error Codes 119 2/C13
6.8 Use of User Supplies Subroutines 119 2/C13
7.0 SAMPLE PROBLEMS 127 2/D7
7.1 Eigenvalue Calculation 127 2/D7
7.2 Time Response Calculation 128 2/D8
7.3 Frequency Response and Power Spectral Density Calculation 131 2/D11
7.4 Root Locus Calculation 133 2/D13
7.5 Sample Problem Output 135 2/E1
Appendix A: QR Eigenvalue Calculation Process 147 2/F9
Appendix B: QR State Space Formulation 161 2/G9
REFERENCES 175 3/A11

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CONTENTS

Page

1.0 SUMMARY 1
2.0 INTRODUCTION 3
3.0 SYMBOLS AND ABBREVIATIONS 4
4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION 8
  4.1 The Basic Polynomial Equation 8
  4.2 Open-Loop and Closed-Loop Systems 10
  4.3 Transfer Function Formation 13
  4.4 Eigenvalue Calculation 15
  4.5 State Space Formulation 16
  4.6 Frequency Response Calculation 17
  4.7 Power Spectral Density Analysis 18
  4.8 Time Response Calculations 21
    4.8.1 Time Response by Method of Residues 21
    4.8.2 Time Response With Tau 23
    4.8.3 Time Response for Forcing Functions that Do Not Have Simple Laplace Transforms 23
  4.9 Unsteady Aerodynamic Options 24
    4.9.1 Frequency Response Analysis 25
    4.9.2 Flutter Analysis 27
5.0 PROGRAM STRUCTURE AND DESCRIPTION 29
  5.1 QR Program Control and Eigenvalue Rooting 29
  5.2 Time Response Calculations 29
  5.3 Frequency Response and PSD Calculations 31
  5.4 Root Locus and V-g Plotting 31
  5.5 Unsteady Airforce Frequency Response and Flutter Calculations 31
6.0 COMPUTER PROGRAM USAGE 32
  6.1 Control Cards 32
  6.2 Resource Estimates 33
  6.3 Card Input Data 34
    6.3.1 Case Initialization 35
    6.3.2 Matrix Element Operations 36
    6.3.3 Roots of Matrix and Root Locus 57
    6.3.4 Transfer Function Formation 69
    6.3.5 Time Response 73
    6.3.6 Frequency Response 79
    6.3.7 Unsteady Aerodynamics Options 84
    6.3.8 Cycling Control Cards and Labeling 94
    6.3.9 Input Card Summary 98

CONTENTS (CONCLUDED)

Page

6.4 Examples of Required Card Sets for Some Typical Problems 107
6.5 Output Data 109
    6.5.1 Printed Output Data 109
    6.5.2 File Output 109
6.6 Restructions 115
6.7 Diagnostics 115
    6.7.1 READTP Error Codes 118
    6.7.2 WRTETP Error Codes 119
6.8 Use of User Supplies Subroutines 119
7.0 SAMPLE PROBLEMS 127
7.1 Eigenvalue Calculation 127
7.2 Time Response Calculation 128
7.3 Frequency Response and Power Spectral Density Calculation 131
7.4 Root Locus Calculation 133
7.5 Sample Problem Output 135
Appendix A: QR Eigenvalue Calculation Process 147
Appendix B: QR State Space Formulation 161
REFERENCES 175

TABLES

Page

1 TAPE92 Matrix Definition for a Frequency Response 110
2 TAPE92 Matrix Definition for a Power SpectralDensity Solution 111
3 TAPE92 Matrix Definition for a Time History Solution 112
4 TAPE92 Matrix Definition for a Root Locus Solution 113
5 TAPE92 Matrix Definition for a Flutter Solution 114

FIGURES

Page

1 Typical Closed-Loop Control Systems 10
2 Typical Open-Loop Control System 11
3 Cancellation of Poles and Zeros 14
4 A One-Minus Cosine Forcing Function 23
5 QR Overlay Structure 30
6 Contents of TAPE92 109
7 Fortran Listing for Subroutine MUSER 120
8 Fortran Listing for Subroutine VUSER 122
9 Fortran Listing for Subroutine USERB 123
10 Fortran Listing for Subroutine ASSIST 124
11 Fortran Listing for Subroutines CLEAR and NEGMAT 125
12 Interface With Subroutines MUSER and VUSER. 125
13 Interface With Subroutine USERSB 126
14 A Simple Feedback Control System 127
15 A Sample Control System for Time Response Calculation 128
16 A Sample Control System for Frequency Response Calculation 131
17 A Sample Control System for Root Locus Calculation 133

1.0 SUMMARY

The program L224 (QR) is primarily a tool for applying classical control systems analysis and synthesis techniques. To make use of the program, the physical system to be analyzed is initially described by a system of simultaneous ordinary linear differential equations with constant coefficients. By manipulating the input matrix of equations, sets of eigenvalues that represent the denominator (poles) and numerator (zeros) of the system transfer function can be calculated. From the transfer functions, studies of time response, frequency response, and root locus may be performed.

The QR program can be divided into five different analyses: eigenvalue and determinant rooting, time response, frequency/PSD response, root locus, and unsteady aerodynamics computations.

Significant program restrictions are:

- The number of eigenvalues for a matrix cannot exceed 140. (Default = 60)
- READTP matrices cannot exceed 70 x 70. (Default 30 x 30)
- Maximum number of repeated poles in the time response calculation is 6
- Maximum number of root locus or flutter eigenvalue points which can be plotted is 2500
- Maximum vector array size is 120
- Maximum number of gains is 48
- Maximum number of phases is 6
- Maximum number of matrix locations into which gain values may be inserted is 8
- Maximum body stations array size is 8
- Maximum forcing functions array size is 12
- Maximum number of reduced frequencies is 70. (Default = 30)

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2.0 INTRODUCTION

The computer program QR (L224) was developed for use as either a stand-alone program, or as a module of the DYLOFLEX program system (ref. 1). QR was designed to meet the DYLOFLEX contract requirements as defined in reference 2. These requrements specify the need for a computer program which can apply classical control systems analysis and synthesis techniques to a set of simultaneous linear differential equations.

The objective of this volume is to aid those persons wishing to use this program. To meet this objective, the following items are discussed:

- Engineering and mathematical equations used to formulate the problem
- Program structure and design
- Guidelines for the execution of the program

Sample problems are also presented to aid the user in the execution of the program.

3.0 SYMBOLS AND ABBREVIATIONS

a, b, c, d, e, f, g, h, I	Dummy coefficients used for the mathematical description of a typical control system.
a_{nm}	Elements of the matrix polynomial of a system with the feedback gain removed.
$[A]$	State variable coefficient matrix.
$[A_o]$	Matrix of coefficients of the zero power of s formed in the QR eigenvalue process.
$[A_1]$	Matrix of coefficients of the first power of s formed in the QR eigenvalue process.
$[A'_o]$	Product of $[A_1]^{-1}\ [A_o]$.
b_{nm}	Elements of the matrix polynomial of a system which are factors of the feedback gain.
b_r	Reference length, meters (in.).
$[B]$	State variable input matrix.
COEF	Leading coefficient of the determinant of the matrix polynomial.
CP	Leading coefficient of the polynomial formed by the system's complex poles.
$C(s)$	A controlled variable.
CZ	Leading coefficient of the polynomial formed by the system's complex zeros.
$[C]$	Output matrix of state variables.
$[C_s]$	Structural damping matrix.
$\{C_3\}$	Generalized forcing function matrix.
$[D]$	Output matrix for inputs.
$e_1(s)$	Feedback signal.

$e_2(s)$	Actuating signal.
$\left[E_1(k)\right]$	Complex airfoce matrix which when multiplied by the freestream dynamic pressure becomes an aerodynamic stiffness matrix.
$\left[E_2(k)\right]$	Complex airforce matrix normalized by $-k^2$, $\left[E_2(k)\right] = -1/k^2\left[E_1(k)\right]$.
f_F	Flutter frequency in hertz.
f_j	Frequency on the j^{th} eigenvalue in hertz.
$f(t)$	Forcing funtion in the time domain.
$f(s)$	Forcing function in the s-plane.
$\{F\}$	Force vector array.
$\left[F_i\right]$	Matrix of coefficients for the i^{th} power of s.
$\left[F_i'\right]$	Matrix of coefficients for the i^{th} power of s with the selected column replaced by the appropriate vector column.
$[F(s)]$	Matrix polynomial in s.
$\|[\overline{F}(s)]\|$	Remaining determinant after the obvious roots have been extracted from the determinant of the matrix polynomial.
$\left[F_p(s)\right]$	Matrix polynomial composed of the systems open loop poles.
$\left[F_z(s)\right]$	Matrix polynomial composed of the systems open loop zeroes.
g	Flutter damping ratio.
$G(s)$	Transfer function of the forward path.
$h(t)$	Impulse response function.
$H(s)$	Transfer function of the feedback path.
$\{ID\}$	Identification array which relates to the y's to the q's and the derivatives of q.

k	Reduced frequency = $\omega b_r / V$.
K	System feedback gain.
$\overline{K}$	Magnitude of a complex gain.
K_{mn}	Residues of the system transfer function.
$[K_s]$	Structural stiffness matrix.
K_1, K_2, K_3	Gust spectrum constants.
L	Gust characteristic length, meters (ft.).
$\mathcal{L}(\ \ )$	Laplace operator.
m	Number of poles.
$\overline{m}_j$	Number of repeated values of the j^{th} pole.
$[M_s]$	Structural inertia matrix.
$[M_1], [M_2], [M_3]$	Generalized structural stiffness damping and inertia matrices.
$[M_4], [M_5]$	Generalized aerodynamic stiffness and damping matrices.
n	Number of zeroes.
P_j	j^{th} pole of a system.
$P(s)$	Polynomial formed from the poles of the system.
$\{q\}$	Column vector of generalized coordinates.
r_j	j^{th} obvious root of the matrix polynomial.
$R(s)$	System input.
s	Laplace variable.
$T(s)$	System transfer function.
$T'(\omega)$	System transfer function multiplied by the square root of the gust spectrum.

$T^{*\prime}(\omega)$	Complex conjugate of $T'(\omega)$.
$\{U_j\}$	Column vector of coefficients for the j^{th} power of s .
$v_g(t)$	Gust velocity, length/sec.
V	Freestream velocity of the airplane, meter/(ft/sec).
V_F	Flutter velocity, meter/sec (ft/sec).
$\{x\}$	Column vector of state variables.
$\{y\}$	Column vector of output variables.
Z_l	l^{th} zero of a system.
$Z(s)$	Polynomial formed from the zeroes of the system.
Γ	Airforce coefficient relating the airforce matrix to a specific flight condition, a non-dimensional factor.
ϵ	Dimensional correction factor.
ζ_j	Damping factor for the j^{th} eigenvalue of the polynomial in s .
λ_j	j^{th} eigenvalue of the polynomial in s .
σ_x	Root mean square value of the gust velocity, meter/sec (ft/sec).
ρ	Density of air, kg-sec^2/meters^4$\left(\text{lb-sec}^2/\text{ft}^4\right)$.
ϕ	Phase angle of the complex gain.
$\Phi(t)$	Wagner indicial lift growth function.
$\Phi_g(\omega)$	Power spectral density of the gust.
$\Phi_r(\omega)$	Power spectral density of the response.
$\Psi(t)$	Küssher indicial lift growth function.
ω	Frequency, rad/sec .

4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION

4.1 THE BASIC POLYNOMIAL EQUATION

The QR program is primarily a tool for applying classical control system analysis and synthesis techniques. The physical system to be analyzed is initially described by a system of simultaneous ordinary linear differential equations with constant coefficients. These equations can then be reduced to a system of simultaneous algebraic equations in the s-plane by use of the Laplace transformation. The set of algebraic equations can be expressed as:

$$\left[\sum_{i=0}^{n} \left[F_i\right] s^i\right] \{q\} = \left\{\sum_{j=0}^{m} \{U_j\}\, s^j\right\} \qquad (1)$$

where:

$\left[F_i\right]$ = matrix of coefficients for the i^{th} power of s

$\{q\}$ = column vector of the system variables transformed into the s-plane

$\{U_j\}$ = column vector of coefficients for the j^{th} power of s which represent the excitation functions transformed into the s-plane

On the left-hand side of equation (1), the set of F matrices shall be referred to as the matrix polynomial in s . The set of U matrices on the right-hand side of equation (1) shall be called the vector polynomial in s .

By manipulating the matrix and vector polynomials, sets of roots can be formed which represent the poles and zeroes of the system transfer functions. These roots are obtained using the QR algorithm (ref. 3). The transfer functions can be used to perform root-locus, time-response, and frequency-response calculations.

In the DYLOFLEX system, the equations of motion for an aircraft expressed in the time domain are:

$$\left[M_1\right]\{q\} + \left[M_2\right]\{\dot{q}\} + \left[M_3\right]\{\ddot{q}\} + \left[M_4\right]\{\dot{q}\}*\Phi(t) + \left[M_5\right]\{\ddot{q}\}*\Phi(t)$$

$$= \{C_3\}\,\dot{v}_g * \psi(t) \qquad (2)$$

where:

$\left[M_1\right], \left[M_2\right], \left[M_3\right]$ = aircraft's generalized stiffness, damping and inertia matrices

$\left[M_4\right], \left[M_5\right]$ = generalized aerodynamic stiffness and damping matrices

$\left\{C_3\right\}$ = generalized forcing function coefficients

$\Phi(t), \Psi(t)$ = Wagner and Küsner indicial lift growth functions

$\left\{q(t)\right\}$ = system's generalized coordinates representing the airplane's rigid and elastic coordinates

$v_g(t)$ = gust velocity

These matrix coefficients are formed by the DYLOFLEX equations of motion program, L217 (ref. 4). Assuming the airplane is initially at rest, equation (2) can be written in the s-plane as

$$\left[\left[M_1\right] + s\left[M_2\right] + s^2\left[M_3\right] + s\left[M_4\right] \mathcal{L}(\Phi) + s^2\left[M_5\right] \mathcal{L}(\Phi)\right]\left\{\mathcal{L}(q)\right\}$$

$$= s\left\{C_3\right\} \mathcal{L}(\psi) \mathcal{L}(v_g) \qquad (3)$$

If an active control system (e.g., yaw damper, stability augmentation system, etc.) is part of the dynamic analysis, then the matrices in equation (3) can be augmented to include the mathematical equations (usually written as polynomials in s) which describe the control system. This task can be accomplished either in the DYLOFLEX equation modifying program L219 (ref. 5), or in QR. The augmented generalized coordinate matrix would include not only the airplane's rigid and elastic coordinates but also the control system variables. The M_1, M_2 and M_3 matrices would be expanded to include the polynomial coefficients needed to define the control system.

The DYLOFLEX equation modifying program, L219, forms the characteristic equation of (3) including Wagner indicial lift growth effects. Using a Wagner indicial lift growth function of the following form:

$$\Phi(t) = 1 - a_1 e^{-\alpha_1 t} - b_1 e^{-\beta_1 t} \qquad (4)$$

the characteristic equation of (3) becomes

$$\begin{aligned}
&\Big[s^4 [M_3] + s^3 \Big[[M_2] + [M_5] + (\alpha_1 + \beta_1)[M_3] - (a_1 + b_1)[M_5]\Big] \\
&\quad + s^2 \Big[[M_1] + [M_4] + (\alpha_1 + \beta_1)\big[[M_2] + [M_5]\big] + \alpha_1 \beta_1 [M_3] \\
&\quad - (a_d + b_1)[M_4] - (a_1\beta_1 + b_1\alpha_1)[M_5]\Big] \\
&\quad + s \Big[(\alpha_1 + \beta_1)\big[[M_1] + [M_4]\big] + \alpha_1\beta_1\big[[M_2] + [M_5]\big] \\
&\quad - (a_1\beta_1 + b_1\alpha_1)[M_4] + \alpha_1\beta_1\big[[M_1] + [M_4]\big]\Big]\Big] \} \mathcal{L}(q) \{ = 0
\end{aligned} \qquad (5)$$

Equation (5) is a fourth order matrix polynomial in s and is of the form of equation (1). The roots of equation (5) will provide an assessment of the airplane stability.

If no lift growth effects are taken into account, the Küssner and Wagner functions are unity and (3) becomes a second order equation in s .

$$\Big[\big[[M_1] + [M_4]\big] + s\big[[M_2] + [M_5]\big] + s^2[M_3]\Big] \} \mathcal{L}(q) \{ = \} C_3 \{ \mathcal{L}(v_g) \qquad (6)$$

For the non-lift-growth case, L219 will form the matrix polynomial and the C_3 matrix to be passed to QR.

4.2 OPEN–LOOP AND CLOSED–LOOP SYSTEMS

Equations describing either an open-or closed-loop system can be placed in the form of equation (1). The purpose of this section will be to show how by proper manipulation of the proper elements in the matrix polynomial, the system characteristics can be ascertained.

A closed-loop control system with negative feedback is shown in figure 1.

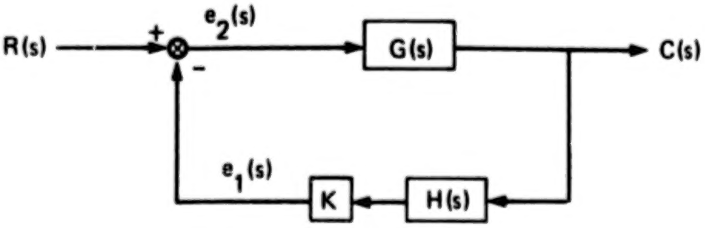


Figure 1. – Typical Closed-Loop Control System

The transfer function for the closed-loop system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + K * G(s) * H(s)} \tag{7}$$

Opening the feedback path of the control system shown in figure 1 would give the open-loop system shown in figure 2.

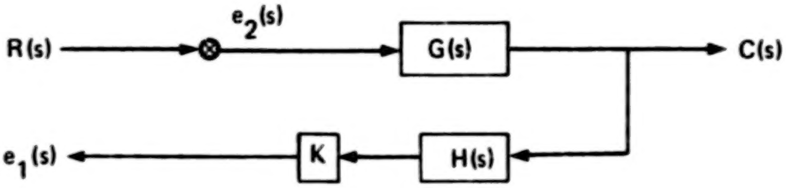


Figure 2. – Typical Open-Loop Control System

The transfer functions for the open-loop system are

$$\frac{C(s)}{R(s)} = G(s)$$

$$\frac{e_1(s)}{R(s)} = K * G(s) * H(s) \tag{8}$$

The characteristic equation for the closed-loop system shown in figure 1, is the demoninator of the system transfer function (equation (7))

$$1 + K * G(s) * H(s) = 0 \tag{9}$$

The roots or poles of the characteristic equation for a closed - loop system indicate the closed-loop stability of that system.

From classical control system analysis (ref. 6), it can be shown that the open-loop poles can be obtained from the roots of equation (9) when the gain, K, is set to zero. The open loop-zeroes are also obtained from equation (9) by setting the gain to infinity. The open-loop transfer function can be expressed as

$$K * G(s) * H(s) = K\left(\frac{Z(s)}{P(s)}\right) \tag{10}$$

where:

$Z(s)$ = polynomial whose roots are open-loop zeroes

$P(s)$ = polynomial whose roots are open-loop poles

and the closed-loop transfer function becomes

$$P(s) + K * Z(s) = 0 \tag{11}$$

The gain K may be expressed as a complex number which contains magnitude and phase information

$$K = \bar{K}\,(\cos\phi + i \sin\phi) \tag{12}$$

where:

$\bar{K}$ = magnitude of the gain

ϕ = phase angle of the gain

By varying the magnitude and phase of K and calculating the roots of equation (11) for each variation, the root locus of the closed-loop system can be obtained.

This is accomplished in QR by manipulating the matrix polynomial of equation (7). For the purpose of discussion, let G(s) and H(s) of figure 1 be represented by the ratio of polynomials in s

$$G(s) = \frac{as + b}{cs^2 + ds + e}$$

$$H(s) = \frac{fs + g}{hs + 1} \tag{13}$$

Using equation (13) and figure 1, the following set of simultaneous equations can be formed

$$\begin{aligned} e_1(s) + e_2(s) &= R(s) \\ (cs^2 + ds + e)\,C(s) - (as + b)\,e_2(s) &= 0 \\ (hs + 1)\,e_1(s) - K(fs + g)\,C(s) &= 0 \end{aligned} \tag{14}$$

In matrix form the equations become:

$$\left[s^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & 0 \\ 0 & -a & d \\ h & 0 & -Kf \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -b & e \\ 1 & 0 & -Kg \end{bmatrix} \right] \begin{Bmatrix} e_1(s) \\ e_2(s) \\ C(s) \end{Bmatrix} = \begin{Bmatrix} R(s) \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

Equation (15) is now of the form of equation (1). The matrix positions of the gain, K, in the matrix polynomial shall be referred to as the gain locations. By first specifying the gain locations in the matrix polynomial of the system, then allowing the gain to vary in magnitude and phase, and finally solving for the roots of the matrix polynomial at each gain variation, the root locus of the system can be obtained. By using the GAINS and the COMPUTE ROOT LOCUS control cards in QR, the aforementioned functions are performed.

4.3 TRANSFER FUNCTION FORMATION

QR can form the transfer function for any system variable, q_k, in equation (1) using the matrix and vector polynomials. From Cramer's rule, q_k can be found by substituting the vectory polynomial into the k^{th} column of the system's matrix polynomial, calculating the determinant of the resulting matrix polynomial, and dividing that determinant by the system's characteristic determinant.

$$q_k = \frac{\left| \left[\sum_{i=0}^{n} \left[F'_i \right] s^i \right] \right|}{\left| \left[\sum_{i=0}^{n} \left[F_i \right] s^i \right] \right|} \qquad (16)$$

In equation (16) the matrices $\left[F'_i \right]$ are polynomial coefficient matrices $\left[F_i \right]$ with the k^{th} column replaced by the $\left\{ U_j \right\}$ vector coefficient matrix.

The characteristic determinant, which is the denominator of (16), can be expressed in terms of its roots and leading coefficient as

$$\left| \left[\sum_{i=0}^{n} \left[F_i \right] s^i \right] \right| = CP \prod_{j=1}^{m} \left(s + P_j \right) \qquad (17)$$

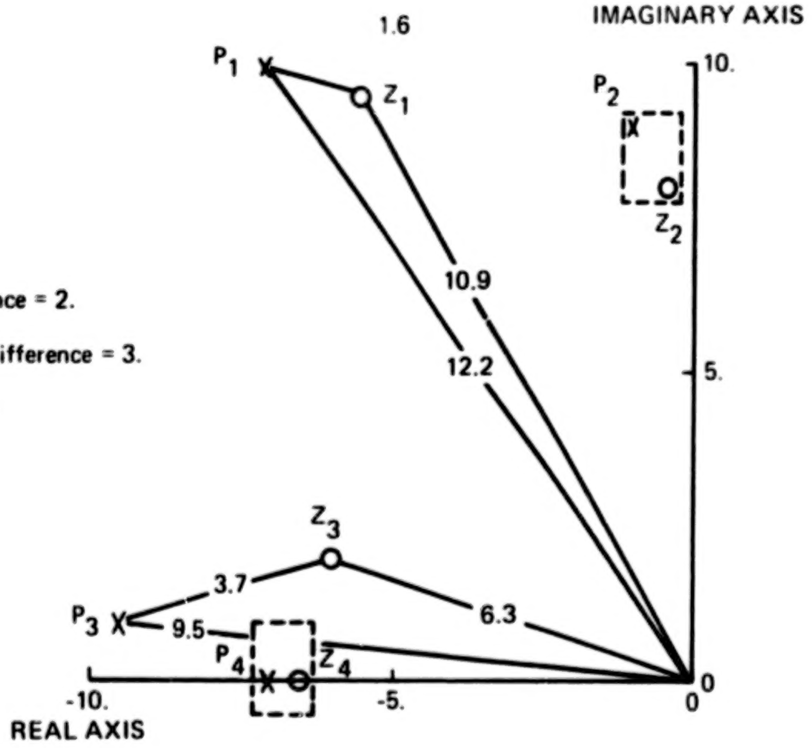
where P_j represents the complex roots of the characteristic equation and CP is the leading coefficient. These roots are the poles of the equation (16). In the QR program, the poles of the transfer function are obtained by using the COMPUTE POLES control card. This control card causes QR to calculate the roots of the matrix polynomial and store these roots as complex poles.

In a similar manner, the numerator of equation (16) can be expressed in terms of its roots and leading coefficient as:

$$\left| \left[\sum_{i=0}^{n} \left[F'_i \right] s^i \right] \right| = CZ \prod_{l=1}^{n} \left(s + Z_l \right) \qquad (18)$$

where Z represents the complex roots of the numerator and CZ is the leading coefficient. These roots are the zeros of equation (16). In the QR program, the zeros of the transfer function are obtained by using the CRAMER REPLACE control card in conjunction with the COMPUTE ZEROS control card.

Once the zeros and poles have been calculated, the transfer function for q_k is obtained from the FORM RATIO OF POLYNOMIALS control card. In the process of forming this ratio, pole-zero pairs may be cancelled (removed from the transfer function) according to the criteria specified by the CANCEL control card. A pole-zero pair is cancelled if either of the following two conditions is met: (1) if the pair can be made to fit within a rectangle of a specified size, or (2) if the ratio of the magnitude of the vector between the pole and zero to the average distance from the origin is less than a specified value. Figure 3 illustrates the cancellation process.



Cancel specifications:

Maximum real difference = 2.

Maximum imaginary difference = 3.

Maximum ratio = .15

- P_2 and Z_2 will cancel by difference test, i.e.,

  REAL (P_2) -REAL $(Z_2) < 2.$

  IMAGINARY (P_2) -IMAGINARY $(Z_2) < 3.$

- P_4 and Z_4 will cancel by difference test
- P_1 and Z_1 will cancel by ratio test, i.e.,

  $1.6/.5(10.9 + 12.2) < .15$

- P_3 and Z_3 will not cancel

Figure 3. – Cancellation of Poles and Zeros

The leading coefficient for the transfer function is formed from CZ and CP and any additional zero and pole gains input by the user using the DC GAIN control card.

$$\text{Leading Coefficient} = (CZ\,/\,CP) * \text{ DC Gain} \tag{19}$$

where DC Gain is the product of zero gains input divided by the product of the pole gains input.

4.4 EIGENVALUE CALCULATION

The QR program obtains the eigenvalues of a matrix polynomial in two steps. First, the order of the determinant of the matrix polynomial is reduced by extracting any common factor (leading coefficient) and all obvious roots in s

$$\left|\left[\sum_{i=0}^{n}\left[F_i\right]s^i\right]\right| = \text{COEF}\prod_{j=1}^{m}\left(s+r_j\right)\ \left|[\bar{F}(s)]\right| \tag{20}$$

where:

COEF = leading coefficient of the matrix polynomial determinant

r_j = obvious roots of the matrix polynomial determinant

$|[F(s)]|$ = remaining determinant after reduction

An initial reduction of the matrix polynomial determinant is achieved using the various mathematical properties of determinants. Appendix A of this document discusses these properties and illustrates the initial reduction process performed in QR.

After completion of the reductions, the remaining matrix polynomial, F(s), will be transformed to eigenvalue form. To accomplish this, the reduced matrix polynomial is expanded into a set of equations which have no derivative of s higher than first order.

$$\left[\left[A_0\right]+s\left[A_1\right]\right]|y|=0 \tag{21}$$

This is accomplished by introducing auxiliary equations which relate the higher orders of s to the first order of s. Multiplying (21) by the inverse of A_1, then the set of equations become

$$\left[\left[A'_0\right]+s[I]\right]|y|=0 \tag{22}$$

The roots of the determinant of expression (22) completes the eigenvalue process. These roots are obtained using the QR algorithm (ref. 3). The reduction of the order F(s) and the normalization of the first order matrix are described in Appendix A.

Combining the obvious roots r_j with those extracted from F(s) and denoting the total set of eigenvalues as λ_j, then equation (20) can be written as

$$\left|\left[\sum_{i=0}^{n}\left[F_i\right]s^i\right]\right| = \text{COEF}\prod_{j=1}^{m'}\left(s+\lambda_j\right) \tag{23}$$

where m' equals the number of obvious roots plus the roots of $\left|[\bar{F}(s)]\right|$.

Several related quantities are calculated from each eigenvalue

$$\begin{aligned} \omega_j &= \left|\lambda_j\right| \\ f_j &= \frac{\omega_j}{2\pi} \\ \zeta_j &= \frac{\text{Real}\left(\lambda_j\right)}{\omega_j} \end{aligned} \tag{24}$$

where:

λ_j = j^{th} complex eigenvalue

ω_j = j^{th} undamped frequency in rad/sec

f_j = j^{th} undamped frequency in hertz

ζ_j = j^{th} damping factor

4.5 STATE SPACE FORMULATION

The QR program has the option of transforming linear, constant coefficient, equations to state variable form. The state variable form has many applications in controls engineering, particularly in optimal control schemes. Using the state variable option, the user can transform the basic set of polynomial equation in s (see fig. 1)

$$[F(s)] \quad \{q\} \quad = \{U(s)\} \tag{25}$$

into two sets of equations

$$\{\dot{x}\} = [A]\{x\} + [B]\{U\} \tag{26}$$

and

$$y = [C]\{x\} + [D]\{U\} \tag{27}$$

where:

$[F(s)]$ = matrix polynomial in s (equation (1)) of size m x m

$\{q\}$ = column vector of variables of size m x 1

$\{U(s)\}$ = vector polynomial in s (equation (1)) of size m x 1

$[A]$ = state variable coefficient matrix of size n x n

$[B]$ = state variable input matrix of size n x m

$[C]$ = output matrix of state variables of size k x m

$[D]$ = output matrix for inputs of size k x m

$\{x\}$ = column vector of state variables of size n x 1

$\{y\}$ = column vector of output variables (composed of all of the q's and possibly some of the derivatives of the q's) of size k x 1

The notation used in equations (26) and (27) is compatible with reference 7.

The method used to transform the set of matrix polynomial equations (25) to first order form is similar to the second part of the eigenvalue solution process as explained in section 4.4 . In the state variable formation, however, the right-hand side, U , must be retained. A detailed description of the reduction process is given in Appendix B.

4.6 FREQUENCY RESPONSE CALCULATION

The QR program calculates the frequency response function for any system variable, q_k, based on the transfer function formulation discussed in section 4.3. The transfer function for q_k can be written as

$$T(s) = \frac{K \prod_{i=1}^{n} \left(s + Z_i\right)}{\prod_{j=1}^{m} \left(s + P_j\right)} \tag{28}$$

where:

Z_i = complex zeros of T(s)

P_j = complex poles of T(s)

K = gain or leading coefficient of T(s)

For equation (28), the frequency response quantities are computed in the following manner. First the substitution $s = j\omega$ is made, then a starting frequency value of the complex operator s is picked such that it is two decades below the smallest eigenvalue. (If there is a pole or zero at the origin, the starting frequency is 0.) The frequency associated with any eigenvalue is always the complex absolute value of it (see eq. (24)).

The magnitude, magnitude expressed in dB, frequency (in hertz) and phase (rad) are calculated for all values of ω selected during the frequency response calculation. The following equations show each of these respective calculations.

$$\text{Magnitude} = \left| T(s) \right|_{s = j\omega} \tag{29}$$

$$\text{Magnitude in dB} = 20 \log_{10} (\text{Magnitude}) \tag{30}$$

$$f = \frac{\omega}{2\pi} \tag{31}$$

$$\text{Phase} = \text{Tan}^{-1} \left(\frac{\text{Imag}\left(T(s)\right) \Big|_{s = j\omega}}{\text{Real}\left(T(s)\right) \Big|_{s = j\omega}} \right) \tag{32}$$

Additional values of frequency are picked in an ascending order such that the magnitude change (equation (30)) from one frequency step to the next frequency step does not exceed 2 dB. In addition, the phase change (equation (32)) from one frequency step to the next frequency step cannot exceed 5 degrees. The last value of frequency used is normally two decades higher than the frequency of the highest frequency eigenvalue. It is possible however, that for certain transfer functions, T(s), that a frequency of this value cannot be attained because the QR program only allows 1000 frequency data points to be selected. This condition is indicated by diagnostic message if it occurs.

Once the arrays of ω, f, gain, dB and phase have been calculated, they are searched for gain and phase margin. Interpolation is provided when requred. Frequency response results can be printed and plotted (Bode and Nyquist plots) when appropriate options are selected by the user.

4.7 POWER SPECTRAL DENSITY ANLAYSIS

Section 4.6 described how a frequency analysis is performed using the poles and zeros that the user designates during his problem solution. All of the frequency response paths in the program can be extended to include power spectral density (PSD) analysis calculations.

When the PSD calculations are made in QR, the transfer function being used is modified by the gust spectrum.

$$T'(\omega) = \left(\Phi_g(\omega)\right)^{1/2} \left(\frac{K \prod_{i=1}^{n} \left(s + Z_i\right)}{\prod_{j=1}^{m} \left(s + P_j\right)} \right) \Bigg|_{s = j\omega} \tag{33}$$

and the power spectral density of the response becomes

$$\Phi_r(\omega) = T'(\omega)T^{*\prime}(\omega) \tag{34}$$

where $T^{*\prime}(\omega)$ is the complex conjugate of $T'(\omega)$. The gust power spectrum is given by the equation

$$\Phi_g(\omega) = \frac{2 K_1 \sigma_x^2 \left(\frac{L}{V}\right) \left(1 + K_2 \left(\frac{L}{V}\right)^2 \omega^2\right)}{\left(1 + K_3 \left(\frac{L}{V}\right)^2 \omega^2\right)^{x/y}} \tag{35}$$

where:

ω = frequency in radians/sec

L = characteristic length

V = freestream velocity of airplane (true airspeed)

σ_x = RMS level of the turbulence

x, y, K_1, K_2, k_3 = spectrum coefficients

The units of $\Phi_g(\omega)$ are $(\text{velocity})^2/\text{hertz}$.

Typical examples of Von Karman forward, lateral, and vertical gust spectrums are

$$\Phi_{u_g}(\omega) = 4\sigma_u^2 \left(L_u/V\right) \frac{1}{\left[1 + 1.7929 \left(L_u/V\right)^2 \omega^2\right]^{5/6}}$$

$$\Phi_{v_g}(\omega) = 2\sigma_v^2 \left(L_v/V\right) \frac{\left[1 + 4.7811 \left(L_v/V\right)^2 \omega^2\right]}{\left[1 + 1.7929 \left(L_v/V\right)^2 \omega^2\right]^{11/6}}$$

$$\Phi_{w_g}(\omega) = 2\sigma_w^2 \left(L_w/V\right) \frac{\left[1 + 4.7811 \left(L_w/V\right)^2 \omega^2\right]}{\left[1 + 1.7929 \left(L_w/V\right)^2 \omega^2\right]^{11/6}}$$

$$\sigma_x^2 = \int_0^\infty \Phi(\omega)\, d\omega$$

Thus, the user might initialize a forward gust with:

$\sigma_u = 1.$ $L_u = 2500.$ $V = 2620.$ $x = 5.$ $y = 6.$ $K_1 = 2.$ $K_2 = 0.$ $K_3 = 1.7929.$

For lateral and vertical gusts the variables might be initialized to

$\sigma = 1.$ $L = 2500.$ $V = 2620.$ $x = 11.$ $y = 6.$ $K_1 = 1.$ $K_2 = 4.7811$ $K_3 = 1.7929.$

The logic used for determining the frequency distribution for the power spectral density calculation is the same as that used in the frequency response calculation discussed in section 4.6. The magnitude of the PSD, the magnitude in dB, frequency in hertz and percent PSD are calculated for all values of ω selected during the PSD response calculation. The following equations show each of these respective calculations:

$$\text{Magnitude} = |T'(\omega)|\ |T^{*\prime}(\omega)| \tag{36}$$

$$\text{Magnitude in dB} = 20 \log_{10} (\text{Magnitude}) \tag{37}$$

$$f = \frac{\omega}{2\pi} \tag{38}$$

$$\text{Percent PSD} = \frac{\int_0^{\omega} T'(\omega)\, T^{*'}(\omega)\, d\omega}{\int_0^{\omega_m} T'(\omega)\, T^{*'}(\omega)\, d\omega} \tag{39}$$

where ω_m = maximum frequency obtained using the criteria discussed in section 4.6.

Once the array of ω, f, magnitude, dB and percent PSD have been calculated, they are printed or plotted depending on the options selected by the user.

4.8 TIME RESPONSE CALCULATIONS

4.8.1 TIME RESPONSE BY METHOD OF RESIDUES

QR computes time responses by determining the inverse Laplace transform of the transfer function

$$T(s) = \frac{K \prod_{i=1}^{n} \left(s + Z_i\right)}{\prod_{j=1}^{m} \left(s + P_j\right)^{\bar{m}_j}} \tag{40}$$

where:

$m \geqslant -n$

Z_i = the complex zeros of T(s)

P_j = the complex poles of T(s)

$\bar{m}_j > 1$ if P_j, is a multiple pole

The inverse transform is obtained by the method of residues.

The transfer function can be written in partial fraction expansion form as:

$$T(s) = \frac{K_{11}}{\left(s + P_1\right)^{\bar{m}_1}} + \frac{K_{12}}{\left(s + P_1\right)^{\bar{m}_1 - 1}} + \ldots + \frac{K_{1\bar{m}_1}}{\left(s + P_1\right)} + \ldots$$

$$+ \frac{K_{m1}}{\left(s + P_m\right)^{\bar{m}_m}} + \frac{K_{m2}}{\left(s + P_m\right)^{\bar{m}_m - 1}} + \ldots + \frac{K_{m\bar{m}_m}}{\left(s + P_m\right)} \tag{41}$$

The residues $K_{11}, K_{12}, \ldots, K_{m1}$ can be found through the usual procedure for single poles. To evaluate K_{11}, both sides of equation (5) are multiplied by $(s + P_1)$. The complex operator s is set to $-P_1$ and the polynomial expansion of the zeros array evaluated with $s = -P_1$.

$$K_{11} = \frac{K \prod_{i=1}^{n} \left(s + Z_i\right) \Big|_{s=-P_1}}{\left(-P_1 + P_2\right)^{\bar{m}_2} \ldots \left(-P_1 + P_m\right)^{\bar{m}_m}} \tag{42}$$

Analogous procedures are used to solve for $K_{12}, \ldots, K_{m1}$. Expression (41) can be written at this point as:

$$\frac{K_{12}}{\left(s + P_1\right)^{\bar{m}_1 - 1}} + \frac{K_{13}}{\left(s + P_1\right)^{\bar{m}_1 - 2}} + \ldots + \frac{K_{1\bar{m}_1}}{\left(s + P_1\right)} +$$

$$\frac{K_{22}}{\left(s + P_2\right)^{\bar{m}_2 - 1}} + \ldots \frac{K_{2\bar{m}_2}}{\left(s + P_2\right)} + \ldots + \frac{K_{m2}}{\left(s + P_m\right)^{\bar{m}_m - 1}} +$$

$$\frac{K_{m\bar{m}_m}}{\left(s + P_m\right)} = T(s) - \frac{K_{11}}{\left(s + P_1\right)^{\bar{m}_1}} - \frac{K_{21}}{\left(s + P_2\right)^{\bar{m}_2}} - \ldots - \frac{K_{m1}}{\left(s + P_m\right)^{\bar{m}_m}} \tag{43}$$

where the residues on the left side are unknown, and those on the right have been evaluated.

For each unknown residue, a complex frequence operator s is selected and substituted into equation (43). The unknown residues are then determined numerically by solving the resulting set of simultaneous linear equations.

After all the residues in equation (41) are known, the time response is:

$$q_k(t) = \left(\frac{K_{11}\, t^{\bar{m}_1 - 1}}{\left(\bar{m}_1 - 1\right)!} + \frac{K_{12}\, t^{\bar{m}_1 - 2}}{\left(\bar{m}_1 - 2\right)!} + \ldots + K_{1\bar{m}_1} \right) e^{-P_1 t}$$

$$+ \ldots + \left(\frac{K_{m1}\, t^{\bar{m}_m - 1}}{\left(\bar{m}_m - 1\right)!} + \ldots K_{m\bar{m}_m} \right) e^{-P_m t} \tag{44}$$

4.8.2 TIME RESPONSE WITH TAU

Designated forcing functions can be turned off after a specified time interval, by using the COMPUTE TIME RESPONSE WITH TAU control cards. To illustrate this QR capability consider the one-minus cosine forcing function shown in figure 4.

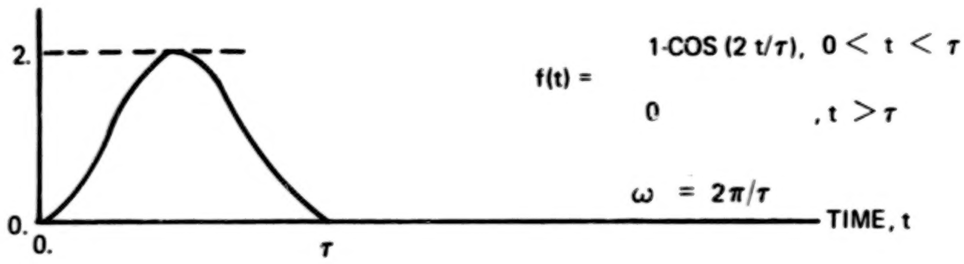


Figure 4. — A One-Minus Cosine Forcing Function

If the one-minus cosine function is represented by the Laplace transform

$$f(s) = \frac{\omega^2}{s\left(s^2 + \omega^2\right)} \qquad (45)$$

the input function is periodic and undamped. However, the desired input is a signal which is zero after the first period.

To achieve this effect, QR solves the time history of the desired response based on the zeros and poles of the periodic forcing function. The time history is then adjusted at $t=\tau$ to reflect the forcing function cutoff.

$$\begin{aligned} q_k(t) &= q_k(t) && t \leq \tau \\ q_k(t) &= q_k(t) - q_k(t-\tau) && \tau \leq t \leq \infty \end{aligned} \qquad (46)$$

where

$q_k(t)$ = time response of the desired output quantity due to the periodic function

4.8.3 TIME RESPONSE FOR FORCING FUNCTIONS THAT DO NOT HAVE SIMPLE LAPLACE TRANSFORMS

The QR program options that perform frequency response, PSD response, and time response calculations all utilize a user-formed ratio of polynomials. It is often desirable to find the time response for one of these transfer functions when an irregular forcing function is applied. The COMPUTE CONVOLUTE control card computes a time history by evaluating the expression:

$$q_k(t) = \int_0^t h(t)\ f(t-\tau)\ d\tau \qquad (47)$$

where:

$h(t)$ = the response of a system excited by an inpulse

$f(t)$ = an aribitrary forcing function described by a set of data points

In QR the transfer function is formed by the **FORM RATIO OF POLYNOMIALS** using the poles and zeros of the system due to a unit impulse input. The inverse Laplace transform is calculated to obtain the impulse response function $h(t)$. The forcing function, $f(t)$, is initialized by using the **FORCING FUNCTION** control card. $f(t)$ should be input to the QR program so that its range is from time equals zero to the final time specified on the **TIME INTERVALS** data card. The time increment input on the **TIME INTERVALS** card should be one-tenth of the period of the highest frequency in the transfer function.

4.9 UNSTEADY AERODYNAMIC OPTIONS

The QR program has the capability of performing analyses upon systems that employ unsteady (frequency dependent) aerodynamics. The complex air force matrices as a function of reduced frequency must be supplied from a source external to QR. The user options include both frequency response analyses (BDOE/NYQUIST) and flutter analyses ($V - \omega$, $V - g$).

QR assumes that the equations of motion with unsteady aerodynamics are in either of two forms:

$$\left[\left[M_s \right] s^2 + \left[C_s \right] s + \left[K_s \right] - \frac{1}{2} \epsilon \rho \ V^2 \left[E_1(k) \right] \right] \{q\} = \{F\} \qquad (48)$$

$$\left[\left[M_s \right] s^2 + \left[C_s \right] s + \left[K_s \right] + \frac{\omega^2}{\Gamma} \left[E_2(k) \right] \right] \{q\} = \{0\} \qquad (49)$$

where:

$[M_s]$	consists primarily of generalized mass and control system elements
$[C_2]$	consists primarily of structural damping and control system elements
$[K_2]$	consists primarily of generalized stiffness and control system elements
$\left[E_1(k)\right]$	is a complex airforce matrix which when multiplied by the freestream dynamic pressure becomes an aerodynamic stiffness matrix
$\left[E_2(k)\right]$	is the complex airforce matrix mormalized by $\left(-k^2\right)\left(\text{i.e.}, \left[E_2(k)\right] = \left[E_1(k)\right] / -k^2\right)$; when multiplied by k^2, $\left[E_2(k)\right]$ becomes an aerodynamic inertia matrix

ρ is the freestream fluid density

V is the freestream velocity (e.g., true airspeed)

ϵ is a dimensional correction factor of value unity unless otherwise specified

Γ is an airforce coefficient relating the airforce matrix to a specific flight condition and to the dimensions of the system (non-dimensionalizing factor)

k is the reduced frequency associated with each airforce matrix $\left(= \omega b_r / V\right)$

ω is the circular frequency of the response

b_r is the reduced frequency reference length

s is the Laplace transform variable ($s = j\omega$)

$\{q\}$ are the generalized coordinates and dependent variables of the system

$\{F\}$ is a force vector array

The QR program has the capability of storing up to three independent inputs. A closed-loop response can be obtained for each column vector individually.

4.9.1 FREQUENCY RESPONSE

The frequency response option for the equations of motion with unsteady aerodynamics is based on equation (48). Since $E_1(k)$ contains frequency dependent aerodynamics, roots for a unique transfer function for the entire frequency spectrum cannot be determined directly from the equations of motion. Consequently, the response must be evaluated at each frequency of interest. The aerodynamic stiffness matrix must correspond to that frequency. If the response is computed for a sufficient number of frequency values, a response in the form of BODE/NYQUIST can be obtained.

The closed-loop response for an independent forcing vector $\{F\}$ is obtained by a classical method of solving a set of non-homogeneous complex linear differential equations. If an open-loop stability analysis is desired for a homogeneous set of equations, a variation of the simultaneous solution approach is employed by QR.

In order to understand how QR treats the unsteady aerodynamic case, it will be useful to review how it handles the non-frequency dependent case. The QR standard method (no unsteady aerodynamics) for computing open-loop poles and zeros follows from equation (11) rewritten

$$F(K, s) = 0 = P(s) + KZ(s) \tag{50}$$

or

$$[F(K, s)] = \left[F_p(s)\right] + K\left[F_z(s)\right] \tag{51}$$

For a single input/single output linear response, the gain constant K must lie solely along a single row or along a single column. The open-loop poles are the roots of $\left[F_p(s)\right]$ and the open-loop zeros are the roots of $\left[F_z(s)\right]$.

$$\left|\left[F(K, s)\right]\right| = \left|\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} + Kb_{1m} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2m} + Kb_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} + Kb_{nm} & \cdots & a_{nn} \end{bmatrix}\right| \quad (52)$$

$$\left|\left[F_p(s)\right]\right| = \left|\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} & \cdots & a_{nn} \end{bmatrix}\right| \quad (53)$$

$$\left|\left[F_z(s)\right]\right| = \left|\begin{bmatrix} a_{11} & a_{12} & \cdots & b_{1m} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & b_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & b_{nm} & \cdots & a_{nn} \end{bmatrix}\right| \quad (54)$$

This method is also valid if all gains are located along one row since

$$\left|\left[A\right]\right| = \left|\left[A\right]^T\right| \quad (55)$$

QR applies Cramer's rule in these computations.

When unsteady aerodynamics are included in the analysis, QR uses a simultaneous equation solution approach which is equivalent to the Cramer's rule approach. If the gains are located along one column, for example

$$\begin{bmatrix} a_{11} & a_{21} & \cdots & \left(a_{1m} + Kb_{1m}\right) & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \left(a_{2m} + Kb_{2m}\right) & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ a_{n1} & a_{n2} & \cdots & \left(a_{nm} + Kb_{nm}\right) & \cdots & a_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{Bmatrix} = \{0\} \qquad (56)$$

QR solves for q_m after transferring all gain elements to the corresponding locations in the right-hand side column vector and setting $s = j\omega$.

$$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1m} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2m} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nm} & \cdots & a_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{Bmatrix} = \begin{Bmatrix} Kb_{1m} \\ Kb_{2m} \\ \cdot \\ \cdot \\ \cdot \\ Kb_{nm} \end{Bmatrix} \qquad (57)$$

Equivalently, by Cramer's rule,

$$q_m = K \left. \frac{Z(s)}{P(s)} \right|_{s = j\omega} \qquad (58)$$

If the gains are located along one row, QR forms the transpose of the characteristic matrix. The gain location elements again become located along one column and the above procedure applies.

4.9.2 FLUTTER ANALYSIS

QR can perform flutter response analyses on homogeneous equations of either of the forms (48) or (49). The latter has been the most frequently used equation. By substituting $-\omega^2 = s^2$, equation (49) can be written

$$\left[\left[\left[M_s\right] - \frac{1}{\Gamma}\left[E_2(k)\right]\right] s^2 + \left[C_s\right] s + \left[K_s\right]\right] \{q\} = \{0\} \qquad (59)$$

Only roots in the proximity of the imaginary axis are valid for resulting flutter analysis interpretations. QR roots the characteristic matrix of equation (48) directly to get the flutter solution.

QR computes the flutter velocity, frequency, and damping values from the flutter roots in the form

$f_F = \mathrm{ABS}(\lambda)/2$ flutter frequency

$\zeta = \mathrm{Re}\,(\lambda)/2\,f_F$ damping ratio

$g = -2\zeta$ flutter damping ratio

$V_F = 2\,b_r f_F/k$ (for small damping)

where k is the reduced frequency associated with the airforce matrices.

5.0 PROGRAM STRUCTURE AND DESCRIPTION

The QR program was written as a series of independent subprograms and then integrated into a single program with several levels of overlays. The QR program can be divided into five different analyses: eigenvalue rooting, time response, frequency/PSD response, root locus, and unsteady aerodynamics. Each major computational task is generally built into one overlay of the program.

Figure 5 shows a block diagram of the overlay structure in QR. The last letter of the overlay block name generally signifies what is actually done during execution. The M in the QRM stands for main executive program; T in QRT stands for time response analysis; R in QRR stands for root locus; U in QRU stands for unsteady aerodynamics. QRF calculates frequency/PSD response.

For detail the task of each overlay see Linear Systems Analysis Program L224 (QR), Volume II (ref. 8).

5.1 QR PROGRAM CONTROL AND EIGENVALUE ROOTING

The QRM, 0, 0 overlay is the QR main program and its purpose is to control the tasks that are undertaken. QR reads a data card from the INPUT file and determines if a valid command has been input. Assuming a valid command has been input, this overlay will do one of two things: (1) branch to another subroutine or position in the main program (QR) where additional input data is read or analytic steps are performed, (2) call another overlay to perform time response, frequency response, root locus plooting, V-g analysis plotting, or unsteady aerodynamics options.

The principal analytic step undertaken in the QRM, 0, 0 overlay is eigenvalue rooting. In addition, this overlay: (1) controls looping for phase gain locus and calculation of open-loop zero and poles; (2) controls looping for altitude and reduced frequencies when performing V-g analysis; and (3) calculates the polynomial coefficients associated with a given set of eigenvalues.

The QRM, 1, 0 overlay calculates constants that are to be used in QR, prints and identification page on the output file (TAPE6), sets up pooled program buffers, and reads the first QR data card from the input file (TAPE5).

The QRM, 7, 0 overlay is the last logical step executed in the QR program. The maximum field length used during the QR execution is calculated and printed on the output file (TAPE6) and the dayfile, the SC4020 plot buffer is filled and emptied onto disk unit 99, local scratch files TAPE90, TAPE91 and TAPE96 are returned, and the CRU cost of the QR execution is calculated.

5.2 TIME RESPONSE CALCULATIONS

Overlay QRT, 2, 0 calculates time response given a polynomial transfer function (Z(s)/P(s)), time intervals, and print/plot instructions. All of this input information is read from data cards or calculated in the QRM, 0, 0 overlay and passed into QRT, 2, 0 by named common.

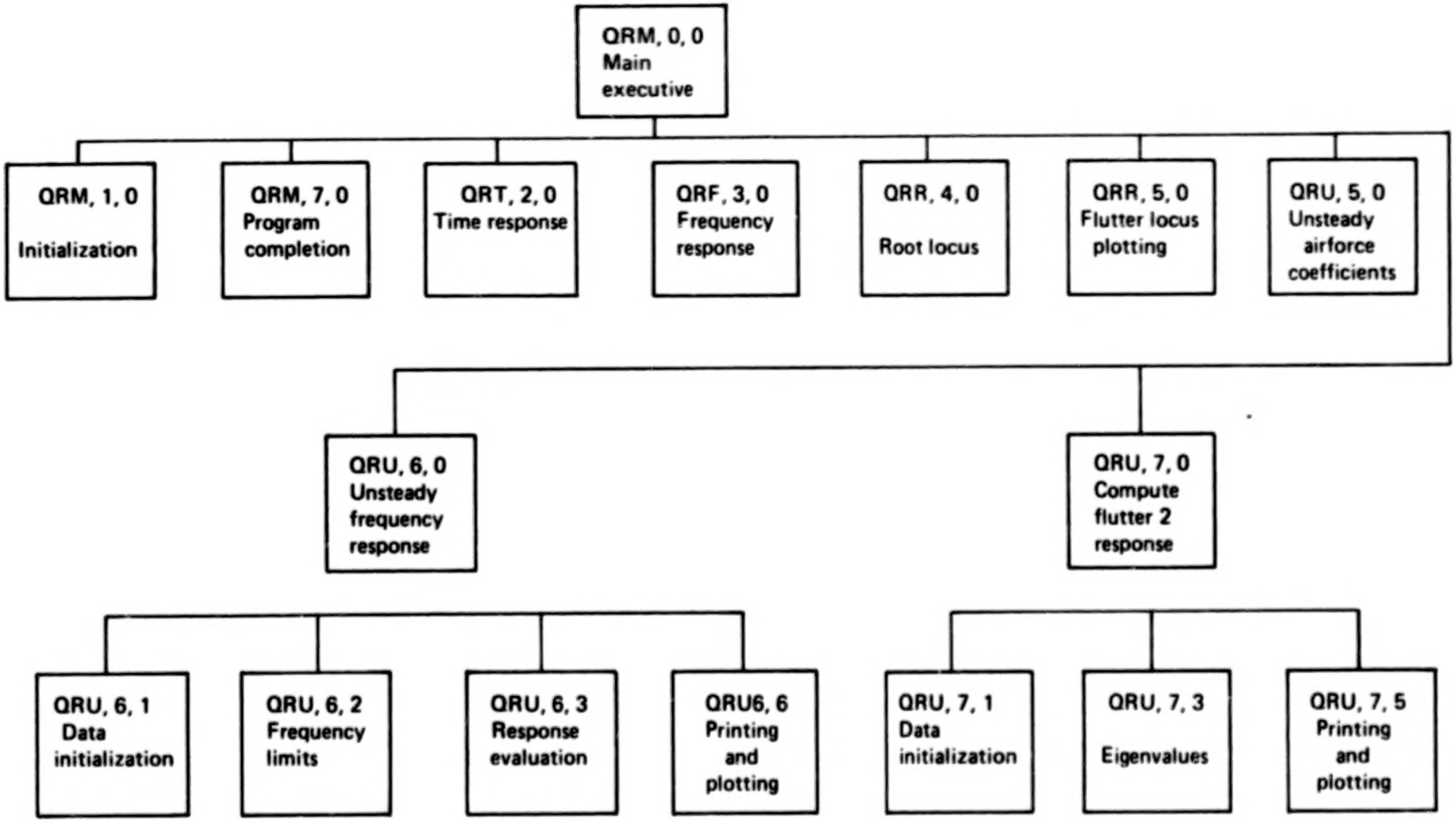

QRM, 0, 0
Main executive
QRM, 1, 0
Initialization
QRM, 7, 0
Program completion
QRT, 2, 0
Time response
QRF, 3, 0
Frequency response
QRR, 4, 0
Root locus
QRR, 5, 0
Flutter locus plotting
QRU, 5, 0
Unsteady airforce coefficients
QRU, 6, 0
Unsteady frequency response
QRU, 7, 0
Compute flutter 2 response
QRU, 6, 1
Data initialization
QRU, 6, 2
Frequency limits
QRU, 6, 3
Response evaluation
QRU6, 6
Printing and plotting
QRU, 7, 1
Data initialization
QRU, 7, 3
Eigenvalues
QRU, 7, 5
Printing and plotting

Figure 5. — QR Overlay Structure

5.3 FREQUENCY RESPONSE AND PSD CALCULATIONS

The QRF, 3, 0 overlay calculates frequency and PSD response given a transfer function (Z(s)/P(s)), gust spectrum, and print/plot instructions. All of this input information is read from data cards or calculated in the QRM, 0, 0 overlay and passed into QRF, 3, 0. by named common.

5.4 ROOT LOCUS AND V-g PLOTTING

Overlay QRR, 4, 0 generates root locus plots given Z(s), P(s), root locus eigenvalues, and plot specifications. All of this input information is read from data cards or calculated in the QRM, 0, 0 overlay and passed into QRR, 4, 0 by named common.

If QR is performing a COMPUTE FLUTTER AND ROOT LOCUS analysis during execution of this overlay, the Z(s) and P(s) complex arrays are written to disk file TAPE96 for eventual processing in overlay QRR, 5, 0.

QRR, 5, 0 generates V-g plots given eigenvalues, k-values, reduced frequency reference length, and plot specifications. All of this input information is read from data cards or calculated in the QRM, 0, 0 overlay and passed into QRR, 5, 0 by named common.

5.5 UNSTEADY AIRFORCE FREQUENCY RESPONSE AND FLUTTER CALCULATIONS

The frequency response and flutter calculations using unsteady aerodynamics are performed by overlays QRU, 5, 0 through QRU, 7, 5. These overlays read the unsteady aerodynamic force matrices from tape, form the equations of motion, and perform the necessary calculations for frequency response or flutter solutions.

6.0 COMPUTER PROGRAM USAGE

The program was designed for use on the CDC 6600. The machine requirements to execute QR are:

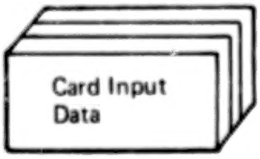
Card Reader	Read control card and card data.
Printer	Print standard output information, optional intermediate calculations, and diagnostic messages.
Disk Storage	All magnetic files not specifically defined as magnetic tapes are assumed to be disk files.
Tape drive	For permanent storage of data. Magnetic files are copied to and from magnetic tapes with control cards before and after program execution.

The program QR is written in FORTRAN and may be compiled with either the RUN or FTN compiler. QR may be executed on either the KRONOS 2.1 or NOS 1.2 operating systems.

6.1 CONTROL CARDS

The following list is a typical set of control cards used to execute QR using the absolute binaries from the program's master tape.

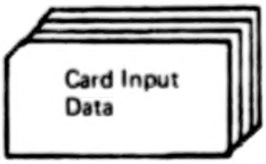
```
Job Card
Account Card
   .
   .
REQUEST (MASTER, F = I, LB = KL, VSN = 66xxxx)   ( Retrieve the program from
REWIND (MASTER)                                   ( its master tape
COPYBF (MASTER, QRM)                              (
COPYBF (MASTER, QRF)                              (
COPY BF (MASTER, QRR)                             (
COPYBF (MASTER, QRT)                              (
COPYBF (MASTER, QRU)                              (
RETURN (MASTER)                                   (
   .
   .                                              {Prepare needed
   .                                              {input files
QRM.                                              } Execute QR
   .                                              {Save desired
   .                                              {output files
EXIT.
DMP (0, field length)
--- End-of-Record
```



```
--- End-of-file
```

The following list is a typical set of control cards used to execute QR using the relocatable binaries from the program's master tape.

```
Job Card
Account Card
      .
      .
      .
REQUEST (MASTER, F = I, LB = KL, VSN = 66 xxxx)    ( Retrieve the program
REWIND (MASTER)                                    { from its master tape
SKIPF (MASTER, 5)                                  {
COPYBF (MASTER, RELQR)                             {
RETURN (MASTER)                                    (
      .                                            (
      .                                            { Retrieve the Dyloflex
      .                                            ( library named DYLIB
LOAD (RELQR, DYLIB)                                { Load QR
NOGO
      .                                            (
      .                                            { Prepare needed input
      .                                            ( files
QRM.                                               { Execute QR
      .                                            (
      .                                            { Save desired output files
      .                                            (
EXIT
DMP (0, field length)
- - - End-of-record
```



```
- - - End-of-file
```

6.2 RESOURCE ESTIMATES

The computer resources used (core requirements, tapes, printed output, central processed seconds, etc.) are a function of the problem size and the program options used.

FIELD LENGTH

The field length (core) required by QR is dependent upon the problem size.

The equation to calculate the field length is

$$\text{Field length} = 105500 + \Big((\text{matrix size}) \times \big((\text{Power of s}) + 1\big)\Big)_8$$

TIME ESTIMATE

The central processing time (cp seconds) required is dependent upon the options chosen and the options chosen and the problem size. The program prints the time required by various operations. It is suggested that 100 seconds be used as a first guess and that it be increased if required.

PRINTED OUTPUT

Output is controlled by the user by use of various print options and selection of the print interval for various outputs.

Pages of output = 2 + number of poles computed/100
+ number of zeros/100
+ number of points for a time history or frequency response/150
+ number of printer plots requested.

Note: This formula is based on a page size of 11 inches by 15 inches and a printing density of 6 lines per inch.

A sample of QR printed output is included in section 7.5.

6.3 CARD INPUT DATA

The QR program accepts data in a menu format, the description of input data is arranged in this format and not in the card set format.

All references to SC4020 plotting in this program are for Boeing use only.

FORMAT OF CARD INPUT DATA

All card data is read in fixed fields (specific column of the cards). The required card columns are defined next to each keyword or variable. The following conventions are used throughout the program:

- All floating point variables are read with format E12.5 or E14.7.
- All integer variables are read with the format I2, I5, or I10.
- All hollerith variables (keywords, etc.) are read with the format 8A10.

All underlined capital characters contained in the keyword/variable field of the input card data must be left justified and punched in the card columns specified in the column field of the input card data. Note that most keywords have an abbreviated form. This form is given in brackets ({ xx }) after each keyword.

For data which is to be read from magnetic file, the user may use tape numbers 1 through 20, except for numbers 5, 6, 7 and 10.

6.3.1 CASE INITIALIZATION

BEGIN Cards

The first card of a case must be a BEGIN control card. The BEGIN card may be used as many many times as required to initialize new cases.

BEGIN card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	BEGIN {B}	8A10	Keyword required to initialize a new execution

BEGIN card-2

This card is optional

COLS.	KEYWORD VARIABLE	FORMAT	DESCRIPTION
1-80	TITLE	8A10	The title input by the user will appear in the printed output. All 80 columns are available for use.

BEGIN card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	MATORD	I5	Number of rows of matrix.
6-10	MATDE	I5	Highest power of s.
11-15	NGR	I5	Maximum number of eigenvalues expected (default = 60).
16-20	NODIAG	I5	If NODIAG = 0, do not print diagnostic. If NODIAG = blank, print diagnostic.
21-25	UNITS	I5	If UNITS = 0, English units. If UNITS = blank, Metric units.

The above cards will print the title of the problem and the size (order) of the matrix input.

Comment Cards

Any card which is distinct from the control cards or a sequence name will be printed with the the following words written to the right:

(THIS IS A COMMENT CARD)

An incorrect control card (e.g., misspelled) will also be interpreted as a comment card.

6.3.2 MATRIX ELEMENT OPERATIONS

This section describes the input cards required to perform the types of matrix operation listed below. Note that the term "matrix polynomial" used throughout this document means a matrix whose elements are coefficients of a polynomial in the Laplace variables (see section 4.1).

• MATRIX	**MATRIX** control card permits values to be read into the matrix polynomial.
• REPLACE	**REPLACE** option permits revisions and additions to be made to the matrix.
• MATRIX COMPLEX	**MATRIX COMPLEX** used to input a complex matrix polynomial.
• REPLACE COMPLEX	**REPLACE COMPLEX** option permits revisions and additions to be made to the complex matrix.
• CONTINUOUS MATRIX	**CONTINUOUS MATRIX** permits card input of a matrix partition for a particular power of s.
• READ MATRIX TAPE	The **READ MATRIX TAPE** option permits magnetic file input of a matrix partition for a particular power of s.
• VECTOR	**VECTOR** option reads data into the vector polynomial.
• REPLACE VECTOR	The **REPLACE VECTOR** option permits revisions and additions to be made to the vector.
• READ VECTOR TAPE	The **READ VECTOR TAPE** option permits magnetic file input of a vector partition for a particular power of s.
• CRAMER REPLACE	The vector polynomial stored in core is inserted into the desired column of the matrix polynomial by use of the CRAMER REPLACE options.
• PRINT	This control card causes the matrix polynomial to be printed.

• PUNCH	This control card causes the matrix polynomial to be punched.
• PRINT PARTIAL MATRIX	This control card allows the user to selectively print portions of his saved matrix.
• SPACE TAPE	This control card enables the user to position a magnetic file.
• DELETE	This control card is used to eliminate the dynamic characteristics of selected elastic modes while retaining their static properties in the equations of motion for purposes of a static elastic solution.
• DELETE AND REDUCE	This control card has the same effect as DELETE card except matrix size is reduced after dynamic characteristics have been eliminated for selected modes.
• TRUNCATE	This control card is used to delete selected rows and columns in matrix equations.
• TRUNCATE AND REDUCE	This control card has the same effect as TRUNCATE card except the matrix size is reduced after the rows and columns have been zeroed.
• BODY STATIONS	This control card is used to read in the body station locations at which the sensor equations are defined.
• SENSOR	SENSOR reads the sensor equations into a specified row of the matrix polynomial.

MATRIX Cards

Each element of the square matrix polynomial has four characteristics; row, column, power of s, and value.

The MATRIX control card permits values to be read into the matrix polynomial. Zero elements do not have to be input. Each value is individually addressed by its row, column, power of s, and value in the matrix polynomial. A maximum of four elements may be input on each card. A field may be left blank on a card if less than four elements are desired.

MATRIX card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	MATRIX {M}	8A10	Keyword indicating that real elements of the matrix polynomial will be read from cards.

MATRIX card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	II	I2	Row	1st Element
3-4	JJ	I2	Column	
5-6	KK	I2	Power of s	
7-20	U(II,JJ,KK)	E14.7	Value	
21-22	II	I2	Row	2nd Element
23-24	JJ	I2	Column	
25-26	KK	I2	Power of s	
27-40	U(II,JJ,KK)	E14.7	Value	
41-42	II	I2	Row	3rd Element
43-44	JJ	I2	Column	
45-46	KK	I2	Power of s	
47-60	U(II,JJ,KK)	E14.7	Value	
61-62	II	I2	Row	4th Element
63-64	JJ	I2	Column	
65-66	KK	I2	Power of s	
67-80	U(II,JJ,KK)	E14.7	Value	

Repeat card-2 as many times as required to define the matrix polynomial. End input of data with a blank card.

If an element address (i.e. row, column and power of s) is listed more than once, the value placed in the matrix polynomial location will be the last value read on the data cards. Cards are read sequentially, and fields are read from left to right.

A blank card after the **MATRIX** card is a legal operation. No data would be read in. However, the matrix polynomial would be set to zero and the null matrices would be printed for each power of s.

REPLACE Cards

The **REPLACE** option permits revisions and additions to be made to the matrix polynomial. If an element had previously been input (e.g., **MATRIX** control card) the **REPLACE** control card could be used to override that element.

REPLACE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	REPLACE (R)	8A10	Keyword indicating that values for real elements of the matrix polynomial which are to be replaced will be read from cards.

REPLACE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	II	I2	Row	1st Element
3-4	JJ	I2	Column	
5-6	KK	I2	Power of s	
7-20	U(II,JJ,KK)	E14.7	Value	
21-22	II	I2	Row	2nd Element
23-24	JJ	I2	Column	
25-26	KK	I2	Power of s	
27-40	U(II,JJ,KK)	E14.7	Value	
41-42	II	I2	Row	3rd Element
43-44	JJ	I2	Column	
45-46	KK	I2	Power of s	
47-60	U(II,JJ,KK)	E14.7	Value	
61-62	II	I2	Row	4th Element
63-64	JJ	I2	Column	
65-66	KK	I2	Power of s	
67-80	U(II,JJ,KK)	E14.7	Value	

Repeat card-2 as many times as required to completely define the matrix elements to be replaced. End input of data with a blank card.

MATRIX COMPLEX Cards

This control card is used to input a complex matrix polynomial. The row, column, power of s, real part, and imaginary part of each complex number must be listed on the data cards. Either one or two complex numbers may be listed per card.

MATRIX COMPLEX card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	MATRIX COMPLEX {MC}	8A10	Keyword indicating that complex elements of the matrix polynomial will be read from cards.

MATRIX COMPLEX card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	II	I2	Row	Complex Element #1
3-4	JJ	I2	Column	
5-6	KK	I2	Power of s	
7-20	$U(II,JJ,KK)_{Re}$	E14.7	Real part	
21-26		6X	Blank	
27-40	$U(II,JJ,KK)_{Im}$	E14.7	Imaginary part	
41-42	II	I2	Row	Complex Element #2
43-44	JJ	I2	Column	
45-46	KK	I2	Power of s	
47-60	$U(II,JJ,KK)_{Re}$	E14.7	Real part	
61-66		6X	Blank	
67-80	$U(II,JJ,KK)_{Im}$	E14.7	Imaginary part	

Repeat card-2 as many times as required to completely define the complex matrix. End input of data with a blank card.

REPLACE COMPLEX Cards

This control card has the same effect as the **REPLACE** control card but utilizes the format of the **MATRIX COMPLEX** options.

REPLACE COMPLEX card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	REPLACE COMPLEX (RC)	8A10	Keyword indicating that values for complex elements of the matrix polynomial which are to be replaced will be read from cards.

REPLACE COMPLEX card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	II	I2	Row	Complex Element #1
3-4	JJ	I2	Column	
5-6	KK	I2	Power of s	
7-20	$U(II,JJ,KK)_{Re}$	E14.7	Real part	
21-26		6X	Blank	
27-40	$U(II,JJ,KK)_{Im}$	E14.7	Imaginary part	
41-42	II	I2	Row	Complex Element #2
43-44	JJ	I2	Column	
45-46	KK	I2	Power of s	
47-60	$U(II,JJ,KK)_{Re}$	E14.7	Real part	
61-66		6X	Blank	
67-80	$U(II,JJ,KK)_{Im}$	E14.7	Imaginary part	

Repeat card-2 as many times as required to completely define the complex matrix elements to be replaced. End input with a blank card.

CONTINUOUS MATRIX Cards

The **CONTINUOUS MATRIX** control card permits input of a matrix partition for a particular power of s. The partition must be square and is read into the upper left-hand corner of the matrix polynomial. This option provides an alternate means for matrix input.

The elements of the partition, including zeros, must be contained on the cards in successive rows. The second row begins in the field following the last element of the first row. A new row does not necessarily start a new card.

CONTINUOUS MATRIX card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	CONTINUOUS MATRIX (CON)	8A10	Keyword indicating that elements of a matrix partition will be read in a continuous manner from cards.

CONTINUOUS MATRIX card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	MTORD	I5	Number of rows of square partition to be read from cards must be less than or equal to the number of rows of the matrix polynomial.
6-10	MTDEG	I5	Power of s corresponding to the partition.

CONTINUOUS MATRIX card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	U(1,1,MTDEG)	E12.5	1st element of partition.
13-24	U(1,2,MTDEG)	E12.5	2nd element of partition.
25-36	U(1,3,MTDEG)	E12.5	3rd element of partition.
37-48	U(1,4,MTDEG)	E12.5	4th element of partition.
49-60	U(1,5,MTDEG)	E12.5	5th element of partition.
61-72	U(1,6,MTDEG)	E12.5	6th element of partition.

Repeat card-3 as many times as required to list all of the elements. A blank card is not needed to terminate the reading of data.

READ MATRIX TAPE Cards

The READ MATRIX TAPE option permits magnetic file input of a matrix partition for a particular power of s. The partition must be square and is read into the upper lef-hand corner of the matrix polynomial.

The maximum permissible matrix size on tape is 70 x 70. The power of s, tape number, and tape positioning are input from data cards.

READ MATRIX TAPE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	READ MATRIX TAPE { RMT }	8A10	Keyword indicating that a matrix will be read from tape.

READ MATRIX TAPE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit. If negative, the tape will be rewound before any file and matrix skipping occurs.
6-10	NFILES	I5	Number of end-of-files to skip. A minus number will backspace the tape that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	MATDG	I5	Power of s of the matrix to be read from tape.
21-25	NPRT	I5	If NPRT = 0, print matrix read from tape. If NPRT = 1, do not print matrix.

Repeat card-2 as many times as required to completely define all the matrix partitions to be read from tape. End reading of matrices by inserting a blank card.

VECTOR Cards

The VECTOR option is similar to the MATRIX option. However, the VECTOR option reads data into the vector polynomial.

VECTOR card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	VECTOR {V}	8A10	Keyword indicating that the elements of a vector polynomial will be read from cards.

VECTOR card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	J1	I2	Row	1st Element
3-4		2X	Blank	
5-6	J2	I2	Power of s	
7-20	VEC(J1,J2)	E14.7	Value	
21-22	J1	I2	Row	2nd Element
23-24		2X	Blank	
25-26	J2	I2	Power of s	
27-40	VEC(J1,J2)	E14.7	Value	
41-42	J1	I2	Row	3rd Element
43-44		2X	Blank	
45-46	J2	I2	Power of s	
47-60	VEC(J1,J2)	E14.7	Value	
61-62	J1	I2	Row	4th Element
63-64		2X	Blank	
65-66	J2	I2	Power of s	
67-80	VEC(J1,J2)	E14.7	Value	

Repeat card-2 as many times as required to completely define the vector polynomial.

The VECTOR cards may be used to input from one to three distinct vectors which can be used for the closed-loop unsteady frequency response analysis. Each vector is stored as a different power of s(i.e. vector 1 is s ** 0, vector 2 is s ** 1 and vector 3 is s ** 2). Using card-2 of the COMPUTE UNSTEADY FREQUENCY RESPONSE card set, the user can extract the particular vector to be used. All vectors used in the unsteady frequency response calculation are considered coefficients of s ** 0.

REPLACE VECTOR Cards

The **REPLACE VECTOR** option permits revisions and additions to be made to the vector. For example, if an element had previously been input by the **VECTOR** sequence, the **REPLACE VECTOR** option could be used to override that element.

REPLACE VECTOR card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	REPLACE VECTOR {RV}	8A10	Keyword indicating that values for elements of the vector polynomial which are to be replaced will be read from cards.

REPLACE VECTOR card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	J1	I2	Row	1st Element
3-4		2X	Blank	
5-6	J2	I2	Power of s	
7-20	VEC(J1,J2)	E14.7	Value	
21-22	J1	I2	Row	2nd Element
23-24		2X	Blank	
25-26	J2	I2	Power of s	
27-40	VEC(J1,J2)	E14.7	Value	
41-42	J1	I2	Row	3rd Element
43-44		2X	Blank	
45-46	J2	I2	Power of s	
47-60	VEC(J1,J2)	E14.7	Value	
61-62	J1	I2	Row	4th Element
63-64		2X	Blank	
65-66	J2	I2	Power of s	
67-80	VEC(J1,J2)	E14.7	Value	

The replace vector polynomial is not initialized to zero.

Repeat card-2 as many times as required to completely define the vector elements to be replaced. End input with a blank card.

READ VECTOR TAPE Cards

The **READ VECTOR TAPE** option permits magnetic file input of a vector partition for a particular power of s. The partition is read into the upper region of the vector polynomial.

READ VECTOR TAPE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	READ VECTOR TAPE (RVT)	8A10	Keyword indicating that a vector will be read from tape.

READ VECTOR TAPE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit. If negative, the tape will be rewound before any file and matrix skipping occurs.
6-10	NFILES	I5	Number of end-of-files to skip. A minus number will backspace the tape that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	MATDG	I5	Power of s of the matrix to be read from tape.
21-25	NPRT	I5	If NPRT = 0, print matrix read from tape. If NPRT = 1, do not print matrix.

Repeat card-2 as many times as required to completely define all vector partitions to be read.

CRAMER REPLACE Cards

The vector polynomial stored in core is inserted into the specified column locations of the matrix polynomial which is also stored in core by use of the **CRAMER REPLACE** option.

CRAMER REPLACE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	CRAMER REPLACE (CR)	8A10	Keyword indicating that the vector polynomial is to be placed in a specified column of the matrix polynomial.

CRAMER REPLACE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	J	I5	The column number of the matrix polynomial into which the vector polynomial is to be placed.
6-10	L	I5	If L = 0, insertion of column is temporary. If L = 1, insertion of column is permanent part of matrix polynomial.

PRINT Card

This control card causes the elements of the matrix polynomial to be printed.

PRINT card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PRINT {P}	8A10	Keyword indicating that the matrix polynomial is to be printed.

PUNCH Card

This control card causes the elements of the matrix polynomial stored in core to be punched on cards in the format of MATRIX card-2 (4(3I2, E14.7)).

PUNCH card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PUNCH {PU}	8A10	Keyword indicating that the matrix polynomial is to be punched on cards.

PRINT PARTIAL MATRIX Cards

This control card allows the user to print portions of the matrix polynomial.

Only non-zero portions of the matrix specified by the user are printed. This format is considerably less voluminous than the sequence initiated by the PRINT control card.

PRINT PARTIAL MATRIX card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PRINT PARTIAL MATRIX {PPM}	8A10	Keyword indicating that only part of the matrix polynomial is to be printed.

PRINT PARTIAL MATRIX card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	I1	I5	Starting row position to be printed.
6-10	I2	I5	Starting column position to be printed.
11-15	I3	I5	Starting power of s to be printed.
16-20	I4	I5	Ending row position to be printed.
21-25	I5	I5	Ending column position to be printed.
26-30	I6	I5	Ending power of s to be printed.

SPACE TAPE Cards

This control card enables the user to move to a different position on a magnetic file.

SPACE TAPE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	SPACE TAPE {ST}	8A10	Keyword indicating that tape positioning data is to be read from cards.

SPACE TAPE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit. If negative the tape will be rewound before any file and matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace the tape that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.

DELETE Cards

This control card is used to eliminate the dynamic characteristics of selected elastic modes in in matrix equations of motion. Static properties of the eliminated modes are retained. To accomplish the elimination, all polynomial coefficients except constant terms in specified columns are set to zero. This procedure eliminates particular modes but retains the residual stiffness of the modes.

DELETE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	DELETE {DE}	8A10	Keyword indicating that the dynamic characteristics of specified elastic modes are to be eliminated.

The following rule must be adhered to when using the DELETE control card:

- The only degrees of freedom which can be deleted are airplane elastic degrees of freedom. The equations of motion are assumed to be of the following form:

$$[A]\ \{x\} = \{F\}$$

where:

$\{F\}$ is the generalized force vector

$\{X\}$ is the column vector of variables

$[A]$ is the equations of motion matrix

DELETE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	M	I5	The number of modes which are to be deleted.

DELETE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	KFIX1	I5	The column number of the 1st mode to be deleted.
6-10	KFIX2	I5	The column number of the 2nd mode to be deleted.
11-15	KFIX3	I5	The column number of the 3rd mode to be deleted.
16-20	KFIX4	I5	The column number of the 4th mode to be deleted.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
76-80	KFIX16	I5	The column number of the 16th mode to be deleted.

Repeat card-3 as many times as required to list all modes to be deleted.

DELETE AND REDUCE Cards

This control card has the same effect as the DELETE control card, except the matrix size is reduced after the dynamic characteristics have been eliminated from the selected modes. The matrix size reduction is equal to the number of modes which are operated upon. After rows and columns are removed, the remaining rows and columns are shifted up and to the left. The remaining modes are renumbered after the reduction.

Note: The DELETE AND REDUCE control card removes the designated rows and columns, and elements of the remaining matrix are modified to account for the static stiffness of the removed degrees of freedom. If the equations of motion represent a partition of a larger system of equations,

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$$

the elements of the A_{11}, A_{12}, and A_{21} partitions are modified. The elements of the A_{22} partition remain unchanged.

DELETE AND REDUCE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	DELETE AND REDUCE (DAR)	8A10	Keyword indicating that the dynamic characteristics of specified modes are to be eliminated and the remaining matrix reduced.

DELETE AND REDUCE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	M	I5	The number of modes which are to be deleted and reduced.

DELETE AND REDUCE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	KFIX1	I5	The column number of the 1st mode to be deleted and reduced.
6-10	KFIX2	I5	The column number of the 2nd mode to be deleted and reduced.
11-15	KFIX3	I5	The column number of the 3rd mode to be deleted and reduced.
16-20	KFIX4	I5	The column number of the 4th mode to be deleted and reduced.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
76-80	KFIX16	I5	The column number of the 16th mode to be deleted and reduced.

Repeat card-3 as many times as required to list all of the column numbers of the modes to be reduced and deleted. A blank card is not necessary to end input.

TRUNCATE Cards

This control card is used to eliminate selected freedoms from the matrix equations. To accomplish the elimination, all polynomial coefficients in the specified rows and columns are set to zero. Following this action, the diagonal elements of the specified rows and columns are set to unity.

The operation performed by this control card allows the user to selectively determine where eigenvalues occur in the matrix equations. For example, if a user has an unstable root, he can arrange a set of control cards which will reduce the matrix, calculate a set of eigenvalues, reduce the matrix, calculate a set of eigenvalues, etc. Inspection of the printout received from this execution should produce a set of eigenvalues that are all stable.

TRUNCATE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TRUNCATE (TRU)	8A10	Keyword indicating that specified rows and columns will be deleted from the matrix polynomial.

TRUNCATE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	M	I5	The number of rows and columns which are to be truncated.

TRUNCATE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	KFIX1	I5	The 1st row and column to be truncated.
6-10	KFIX2	I5	The 2nd row and column to be truncated.
11-15	KFIX3	I5	The 3rd row and column to be truncated.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
76-80	KFIX16	I5	The 16th row and column to be truncated.

Repeat card-3 as many times as required to list all of the row and column numbers of freedoms to be eliminated.

TRUNCATE AND REDUCE Cards

This control card has the same effect as the TRUNCATE card, except the matrix size is reduced after the rows and columns have been zeroed. The matrix size reduction is equal to the number of rows and columns which are truncated. After the rows and columns are zeroed, the remaining rows and columns are shifted up and to the left. The remaining modes are renumbered after reduction.

TRUNCATE AND REDUCE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TRUNCATE AND REDUCE {TAR}	8A10	Keyword indicating that specified rows and columns will be deleted from the matrix polyn-mial and the remaining matrix will be reduced.

TRUNCATE AND REDUCE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	M	I5	The number of rows and columns which are to be truncated and reduced.

TRUNCATE AND REDUCE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	KFIX1	I5	The 1st row and column to be truncated and reduced.
6-10	KFIX2	I5	The 2nd row and column to be truncated and reduced.
11-15	KFIX3	I5	The 3rd row and column to be truncated and reduced.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
76-80	KFIX16	I5	The 16th row and column to be truncated and reduced

Repeat card-3 as many times as required to list all of the row and column numbers of freedoms to be eliminated.

AUTOMATIC INPUT OF SENSOR DATA

The next two sets of control cards, **BODY STATIONS** and **SENSOR**, allow the user to automatically define the sensor equations needed for the desired control system modeling. A matrix (or set) of sensor equations are read from magnetic file. Using the **BODY STATIONS** cards, the user establishes a correlation between the row of the sensor data input matrix and a body station (x reference axis system) location. The sensor body station location required for the particular control system to be analyzed is defined by the user on **SENSOR** card-3. Using the body station-sensor equation correlation defined earlier, the QR program interpolates for the sensor equations at the required body station location. The interpolated sensor equation is then placed in matrix polynomial as indicated on SENSOR card-4.

BODY STATION Cards

This control card defines the body stations at which the incoming sensor equations are defined.

BODY STATIONS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	BODY STATIONS (BS)	8A10	Keyword indicating that the body station locations for the sensor equations being input will be read from cards.

BODY STATIONS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NBS	I5	Number of body stations to be read($\leq$ 8)

BODY STATIONS card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	X(1)	E12.5	Body station location of the 1st sensor equation being read from tape.
13-24	X(2)	E12.5	Body station location of the 2nd sensor equation being read from tape.
.	.	.	.
.	.	.	.
.	.	.	.
61-72	X(6)	E12.5	Body station location of the 6th sensor equation being read from tape.

Repeat card-3 as many times as required to read the specified number of body stations indicated by NBS on card-2. Omit card-3 if NBS = 0.

SENSOR Cards

The SENSOR control card set defines the sensor equations which are to be read into the matrix polynomial. The row and column size of the matrix and vector polynomial are not increased.

SENSOR card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	SENSOR (SE)	8A10	Keyword indicating that sensor equations are to be read from tape.

SENSOR card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit. If negative, the tape will be rewound before any file or matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace the tape that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	ICYCLE	I5	The number of sensor equations to generate. If ICYCLE = 0 or 1, only one sensor equation is generated.

SENSOR card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	BS	E12.5	The body station location of the sensor equation which is to be placed in the matrix polynomial.

SENSOR card-4

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	IQROW	I5	Row Position.
6-10	IQPOW	I5	Power of s.
11-15	ILIM	I5	If ILIM = 0, all columns are read from tape. If ILIM ≠ 0, columns 1 through value of ILIM are read.

Repeat card-3 and card-4 as many times as required to generate the number of sensor equations as indicated by ICYCLE (card-2).

6.3.3 ROOTS OF MATRIX AND ROOT LOCUS

The entire computing program depends upon the QR algorithm to obtain roots of a matrix polynomial. The root locus portion of the program uses the QR algorithm directly to obtain roots for the specified gain values.

• COMPUTE	Computes root of matrix currently in core and prints results.
• COMPUTE POLES	Same as COMPUTE except roots are saved in pole (denominator) array.
• COMPUTE ZEROS	Same as COMPUTE except roots are saved in zero (numerator) array.
• COMPUTE LOCUS	Same as COMPUTE except roots are saved for plotting purposes.
• GAIN	The GAINS card permits input of gains values, phase values, and matrix polynomial locations for the gains which are to be used for the root locus calculations.
• COMPUTE ROOT LOCUS	This control card causes the computer to enter the root locus calculation sequence.
• COMPUTE ROOT LOCUS WITH PREVIOUS POLES	This control card initiates the root locus calculation sequence using poles previously calculated by either a COMPUTE POLES card or COMPUTE ROOT LOCUS card.
• COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS	This card initiates the root locus calculation sequence using zeros previously calculated by either a COMPUTE ZEROS card or COMPUTE ROOT LOCUS card.

• COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS	This card initiates the root locus calculation sequence using poles and zeros previously calculated by use of the **COMPUTE POLES**, **COMPUTE ZEROS**, and/or **COMPUTE ROOT LOCUS** control card.
• FORM POLYNOMIAL	This control card causes the characteristic polynomial to be be formed for each set of roots which is computed.
• NO FORM POLYNOMIAL	This control card negates the effect of the **FORM POLYNOMIAL** control card.
• PLOT ROOT LOCUS	This car permits specification of the root locus plots.
• EXECUTE PLOTS	This card permits specified sets of roots to be plotted on in the s-plane.
• EXECUTE PLOTS WITH POLES	This card has the same effect as **EXECUTE PLOTS** except poles are plotted as x's in addition to the roots which are plotted as dots.
• EXECUTE PLOTS WITH ZEROS	This card is similar to the **EXECUTE PLOTS WITH POLES** control card. Zeros are plotted with o's in addition to the roots which are plotted as dots.
• EXECUTE PLOTS WITH POLES AND ZEROS	This card combines the features of the above two cards to produce plots with roots as dots, poles as x's, and zeros as o's.
• STATE	This card transforms matrix polynomial equation to state variable form and calculates roots.

The control cards **COMPUTE POLES** and **COMPUTE ZEROS** are employed to establish transfer functions for use by the time response and frequency response portions of the program. A typical procedure to follow for using these control cards is as follows:

- Form a matrix which represents the characteristics of a system
- Use the **COMPUTE POLES** control card
- Use Cramer's rule to establish a matrix which represents the numerator of one of the system variables
- Use the **COMPUTE ZEROS** control card

Both the poles and zeros are saved permanently within the computer. Both arrays may be changed independently by the subsequent use of the **COMPUTE POLES** and **COMPUTE ZEROS** control cards. Manipulations of the poles and zeros are described in the Transfer Function Formation section, 6.3.4.

Root locus calculations are initiated by a COMPUTE ROOT LOCUS control card, or one of the variations of the COMPUTE ROOT LOCUS control card. The root locus calculation sequence is as follows:

- Compute open-loop poles and zeros based on gain locations specified by the user
- Cancel nearly equal pole/zero pairs which fall into tolerances specified by the user
- Compute an open-loop frequency response (Nyquist and Bode)
- Compute roots for all specified gain values and phase angles
- Plot the results in root locus, Bode, and Nyquist formats with the printout (printer-plotter)

The cancellation of poles and zeros causes a significant reduction in central processor time. For most development work, the cancellation tolerances should be fairly broad in order to coserve machine time and to restrict the printout to only the roots of significant interest. Small or zero cancellation limits should be used for initial runs and final runs involved in the development of a system. Users will be able to readily assess the trades involved because central processor time is printed with each set of roots.

The QR program calculates open-loop poles and zeros as follows:

1. The program searches all gain locations to determine if they are along one row. If the locations are not along one row, the program goes to step no. 7.
2. The program sets all elements on the gain row to zero except elements which correspond to the gain locations.
3. Roots of the modified matrix are computed and stored in the zero array.
4. The matrix is restored to its original form.
5. All elements corresponding to the gain locations are set equal to zero.
6. Roots of the modified matrices are computed and stored in the pole array. The program exits from the zero and pole formation operation.
7. The program searches all gain locations to determine if they are along one column. If the gain locations are not along one column, go to step no. 9.
8. The program sets all elements on the gain column to zero except elements which correspond to the gain location. Go to step no. 3.
9. The following message is printed: "ZEROS CANNOT BE FOUND".

COMPUTE Card

This control card causes roots of the current matrix to be calculated and printed. None of the roots are saved on file.

COMPUTE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE {CO}	8A10	Keyword indicating that the roots of the current matrix polynomial will be calculated and printed.

COMPUTE POLES Card

This control card has the same effect as the COMPUTE control card except that the roots are saved in the pole (denominator) array.

COMPUTE POLES card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE POLES {CP}	8A10	Keyword indicating that the roots of the current matrix polynomial will be calculated,printed and stored as poles.

COMPUTE ZEROS Card

This control card has the same effect as the COMPUTE control card except the roots are saved in the zero (numerator) array.

COMPUTE ZEROS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE ZEROS {CZ}	8A10	Keyword indicating that the roots of the current matrix polynomial will be calculated,printed and stored as zeros.

COMPUTE LOCUS Card

This control card has the same effect as the COMPUTE control card except the roots are save saved in the plot storage file. Many sets of roots can be stored on the plot file. Each appearance of the COMPUTE LOCUS adds a set of roots to the file. Plots can be formed from the stored roots by use of an EXECUTE PLOTS control card described later. Roots saved on the plot file are retained until destroyed by the following control cards: EXECUTE PLOTS, COMPUTE ROOT LOCUS, and BEGIN.

COMPUTE LOCUS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE LOCUS {CL}	8A10	Keyword indicating that the roots of the current matrix polynomial will be calculated,printed and saved for plotting.

GAINS Cards

The GAIN card set permits input of gain values, phase values, and gain locations. The gain and and phase values are used during the COMPUTE ROOT LOCUS sequence to generate a root locus.

GAINS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	GAINS {G}	8A10	Keyword indicating gain values, phase values and gain locations will be read from cards.

GAINS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NGAIN	I5	The number of gains (default = 48).
6-10	NPHASE	I5	The number of phases (default = 6).

GAINS card-3

Omit this card if NGAIN = 0 (card-2).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GAIN(1)	E12.5	1st GAIN value.
13-24	GAIN(2)	E12.5	2nd GAIN value.
25-36	GAIN(3)	E12.5	3rd GAIN value.
37-48	GAIN(4)	E12.5	4th GAIN value.
49-60	GAIN(5)	E12.5	5th GAIN value.
61-72	GAIN(6)	E12.5	6th GAIN value.

Repeat card-3 as many times as necessary to list all gain values as indicated by NGAIN on card-2.

GAINS card-4

Omit this card if NPHASE = 0 (card-2).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	PHASE(1)	E12.5	1st PHASE value. (degree)
13-24	PHASE(2)	E12.5	2nd PHASE value.
25-36	PHASE(3)	E12.5	3rd PHASE value.
37-48	PHASE(4)	E12.5	4th PHASE value.
49-60	PHASE(5)	E12.5	5th PHASE value.
61-72	PHASE(6)	E12.5	6th PHASE value.

Repeat card-4 as many times as necessary to list all phase values as indicated by NPHASE on card-2.

GAINS card-5

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NPLOC	I5	The number of gain locations (default = 8)

GAINS card-6

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION	
1-2	IRCM(1)	I2	Row number	1st gain locations
3-4	IRCM(2)	I2	Column number	
5-6	IRCM(3)	I2	Power of s	
7-20		14X	Blank	
21-22	IRCM(4)	I2	Row number	2nd gain locations
23-24	IRCM(5)	I2	Column number	
25-26	IRCM(6)	I2	Power of s	
27-40		14X	Blank	
41-42	IRCM(7)	I2	Row number	3rd gain locations
43-44	IRCM(8)	I2	Column number	
45-46	IRCM(9)	I2	Power of s	
47-60		14X	Blank	
61-62	IRCM(10)	I2	Row number	4th gain locations
63-64	IRCM(11)	I2	Column number	
65-66	IRCM(12)	I2	Power of s	
67-80		14X	Blank	

Repeat card-6 as many times as necessary to list all the gain locations.

The GAINS card set can also be used to specify the gain locations for the calculation of an open-loop unsteady frequency response.

COMPUTE ROOT LOCUS Card

This control card causes the computer to enter the root locus sequence. Root locus and frequency response calculations are computed, printed, and plotted according to the options specified by the PLOT ROOT LOCUS card set.

COMPUTE ROOT LOCUS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE ROOT LOCUS (CRL)	8A10	Keyword indicating that root locus calculations are to be performed.

Open-loop zeros and poles are computed with respect to the gain locations which have geen specified in the GAIN control card set.

Pole and zero pairs which fall into the tolerances specified by the CANCEL control card are cancelled and printed. The remaining poles and zeros are printed.

An open-loop frequency response is computed unless suppressed by the FREQUENCY RESPONSE control card.

COMPUTE ROOT LOCUS WITH PREVIOUS POLES Card

Use of the COMPUTE ROOT LOCUS WITH PREVIOUS POLES control card pemorms a root locus calcuation using poles previously computed by either a COMPUTE POLES control card or a COMPUTE ROOT LOCUS control card.

COMPUTE ROOT LOCUS WITH PREVIOUS POLES card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE ROOT LOCUS WITH PREVIOUS POLES (CRLWPP)	8A10	Keyword indicating that root locus calculations are to be performed using previously calculated poles.

COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS Card

Use of the COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS control card performs a root locus calculation using zeros previously computed by either a COMPUTE ZEROS control card or a COMPUTE ROOT LOCUS control card.

COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS (CRLWPZ)	8A10	Keyword indicating that root locus calculations are to be performed using previously calculated zeros.

COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS Card

Use of the COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS control card performs a root locus calculation using poles and zeros previously calculated by use of the COMPUTE POLES, COMPUTE ZEROS, and/or COMPUTE ROOT LOCUS control cards.

COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS (CRLWPPAZ)	8A10	Keyword indicating that root locus calculations are to be performed using previously calculated poles and zeros.

FORM POLYNOMIAL Card

This control card causes the characteristic polynomial to be formed for each set of roots which is computed. Once this option is selected, a characteristic polynomial is formed each time roots are calculated by a COMPUTE, COMPUTE ZEROS, COMPUTE POLES, etc.

FORM POLYNOMIAL card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	FORM POLYNOMIAL (FP)	8A10	Keyword indicating that a polynomial will be formed for each set of roots which are computed.

NO FORM POLYNOMIAL Card

This control card negates the effect of the FORM POLYNOMIAL control card. The QR program does not form any characteristic polynomial until the FORM POLYNOMIAL card appears in the input stream.

NO FORM POLYNOMIAL card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	NO FORM POLYNOMIAL {NFP}	8A10	Keyword indicating that no polynomial will be formed for any computed roots. (program default)

PLOT ROOT LOCUS Cards

This control card permits specification for root locus plotting to be read. The user must specify the number of regions of the s-plane to be plotted and the boundaries of these regions. Plots may be done by use of the printer-plotter.

PLOT ROOT LOCUS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PLOT ROOT LOCUS {PRL}	8A10	Keyword indicating that plotting of root locus results is to be done.

PLOT ROOT LOCUS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NPLOT	I5	Number of regions of s-plane to plot. (maximum = 6). (default=0) If the number of region is zero suppress root locus plotting (value assigned automatically by program if not specified by the user).
6-10	NPHA	I5	If NPHA = 0, each change in phase will start a new plot. If NPHA = 1, all phase angles will be plotted on the same plot.
11-15	IRLPF	I5	If IRPLF = 2, only plots from the printer-plotter.

PLOT ROOT LOCUS card-3

Omit card-3 if NPLOT = 0 (card-2)

The regions of the s-plane should be square, i.e., the distance from the upper limit to the lower limit is equal on the x and y axes. For printer-plotter purposes, the vertical distance between the upper and lower limits should be evenly divisible by 5.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	XLFT	E12.5	Left limit of X axis.
13-24	XRT	E12.5	Right limit of X axis.
25-36	YTOP	E12.5	Upper limit of Y axis.
37-48	YBOT	E12.5	Lower limit of Y axis.

Repeat card-3 as many times as necessary to list all of the regions.

EXECUTE PLOTS Card

This control card permits specified sets of roots to be plotted in the s-plane according to the specifications listed in the **PLOT ROOT LOCUS** control card set. The plotting symbol is a dot. This option should not be confused with the automatic plotting of root loci initiated by the **COMPUTE ROOT LOCUS** control card. The roots to be plotted must have been previously formed by a **COMPUTE LOCUS** control card. Printer-plotter plots are generated.

EXECUTE PLOTS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	EXECUTE PLOTS (EP)	8A10	Keyword indicating that that sets of roots are to be plotted in the s-plane according to the specifications on the PLOT ROOT LOCUS control cards.

EXECUTE PLOTS WITH POLES Card

This control card has the same effect as **EXECUTE PLOTS** except poles are plotted with x's in addition to the roots which are plotted with dots. The poles can be formed by the control cards **COMPUTE POLES** or **COMPUTE ROOT LOCUS**.

EXECUTE PLOTS WITH POLES card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	EXECUTE PLOTS WITH POLES (EPWP)	8A10	Keyword indicating that sets of roots and poles (marked with x's) are to be plotted in the s-plane according to the specifications on the PLOT ROOT LOCUS control cards.

EXECUTE PLOTS WITH ZEROS Card

This control card is similar to the EXECUTE PLOTS WITH POLES control card. However, zeros are plotted with o's in addition to the roots which are plotted with dots. The zeros can be formed by the control cards COMPUTE ZEROS or COMPUTE ROOT LOCUS.

EXECUTE PLOTS WITH ZEROS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	EXECUTE PLOTS WITH ZEROS {EPWZ}	8A10	Keyword indicating that sets of roots and zeros (marked with o's) are to be plotted in the s-plane according to the specifications on the PLOT ROOT LOCUS control cards.

EXECUTE PLOTS WITH POLES AND ZEROS Card

This control card combines the features of the above two cards to produce plots with roots as dots, poles as x's and zeros as o's.

EXECUTE PLOTS WITH POLES AND ZEROS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	EXECUTE PLOTS WITH POLES AND ZEROS {EPWPAZ}	8A10	Keyword indicating that sets of roots, poles (marked with x's), and zeros (marked with o's) are to be plotted in the s-plane according to the specifications on the PLOT ROOT LOCUS control cards.

STATE Card

This control card transforms the matrix polynomial equation to state variable form. The roots are calculated and printed.

STATE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	STATE {STA}	8A10	Keyword indicating that the set of simultaneous equations defined in the s-plane are placed in state variable form.

6.3.4 TRANSFER FUNCTION FORMATION

Transfer functions are ratios of factored polynomials in the Laplace operator, s. The roots of of the denominator and numerator polynomials are called poles and zeros, respectively. The user is responsible for performing manipulations (e.g., Cramer's rule) to set the matrices up such that roots of one matrix polynomial are poles, and roots of another matrix are zeros. Formation of the poles and zeros is discussed in the Roots of Matrix and Root Locus section, 6.3.3.

• CANCEL	The CANCEL card causes poles and zeros to be eliminated.
• DC GAIN	The DC GAIN card permits the user to add scale factor to the transfer functions formed by the COMPUTE ROOT LOCUS and FORM RATIO OF POLYNOMIALS control card.
• INTEGRATE	This card permits poles to be added at the origin of the transfer functions formed by the FORM RATIO OF POLYNOMIALS control card.
• DIFFERENTIATE	This card is similar to INTEGRATE control card except zeros are added at the origin of the transfer function.
• FORM RATIO OF POLYNOMIALS	This card causes formation of a transfer function from the poles and zeros which have been previously formed by COMPUTE POLES and COMPUTE ZEROS control cards.

In the transfer function forming process, the poles and zeros are assigned to special arrays. The leading coefficient of the pole array is normalized to unity. Also, provisions are made for the following:

- Cancellation of pole/zero pairs which are equal or nearly equal (see the control card CANCEL)
- Adjusting the transfer function gain (see the control card DC GAIN)
- Adding poles or zeros to the origin (see the control cards INTEGRATE and DIFFERENTIATE)

Note that a transfer function is also formed automatically within the COMPUTE ROOT LOCUS sequence. The process of forming the transfer function is the same as described above with one exception; namely, poles and zeros are not added at the origin, even if the control cards INTEGRATE or DIFFERENTIATE have been used.

CANCEL Cards

The CANCEL control card causes poles and zeros to be eliminated if a pole/zero pair is sufficiently close on the s-plane. All combinations of poles and zeros are tested to determine the cancelable pairs. The cancellation limits are also used in the time response option.

CANCEL card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	CANCEL (C)	8A10	Keyword indicating that poles and zeroes will be eliminated in the transfer function formation according to the specifications given.

CANCEL card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	XREAL	E12.5	The maximum difference between the real values of poles and zeros allowed for cancellation of pole-zero pairs. (Default = 0)
13-24	XIMAG	E12.5	The maximum difference between the imaginary values of pole and zeros allowed for cancellation of pole-zero pairs. (Default = 0)
25-36	CRATIO	E12.5	The maximum magnitude ratio allowed for canellation of pole-zero pairs. (Default = 0)

DC GAIN Cards

The DC GAIN control card permits the user to add scale factors to the transfer functions formed by the COMPUTE ROOT LOCUS and FORM RATIO OF POLYNOMIALS control cards.

A composite gain is formed by dividing the product of the zero gains by the product of the pole gains. This composite gain multiplies the transfer function gain. An example of a use of the DC GAIN control card would be the use of one zero gain equal to 57.3. This gain would convert an output in radian units to degrees units. If the DC GAIN control card is not used, the composite gain is set equal to 1.

DC GAIN card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	DC GAIN (DCG)	8A10	Keyword indicating that scale factors used to modify the transfer function are to be read from cards.

DC GAIN card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NGAINP	I5	Number of pole (denominator) gains (default = 6).
6-10	NGAINZ	I5	Number of zero (numerator) gains (default = 6).

DC GAIN card-3

Omit card-3 if NGAINP = 0 (card-2)

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GAINP(1)	E12.5	Value of gain for the 1st pole.
13-24	GAINP(2)	E12.5	Value of gain for the 2nd pole.
25-36	GAINP(3)	E12.5	Value of gain for the 3rd pole.
37-48	GAINP(4)	E12.5	Value of gain for the 4th pole.
44-60	GAINP(5)	E12.5	Value of gain for the 5th pole.
61-72	GAINP(6)	E12.5	Value of gain for the 6th pole.

DC GAIN card-4

Omit card-4 if NGAINZ = 0 (card-2)

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GAINZ(1)	E12.5	Value of gain for the 1st zero.
13-24	GAINZ(2)	E12.5	Value of gain for the 2nd zero.
25-36	GAINZ(3)	E12.5	Value of gain for the 3rd zero.
37-48	GAINZ(4)	E12.5	Value of gain for the 4th zero.
49-60	GAINZ(5)	E12.5	Value of gain for the 5th zero.
61-72	GAINZ(6)	E12.5	Value of gain for the 6th zero.

INTEGRATE Cards

This control card permits poles to be added at the origin of the transfer functions formed by the **FORM RATIO OF POLYNOMIALS** control card. The number of integrations which are specified are equal to the number of poles added at the origin. This control card can be used to introduce a step function (1/s) to the transfer function. If the **INTEGRATE** control card is not used, the number of integrations is assumed to be zero.

INTEGRATE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	INTEGRATE (I)	8A10	Keyword indicating that poles are to be added to the origin of the transfer function.

INTEGRATE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NINTEG	I5	Number of integrations.

DIFFERENTIATE Cards

This control card is similar to the **INTEGRATE** control card except zeros are added at the origin of the transfer function. This control card is normally used to differentiate an output variable. If the **DIFFERENTIATE** control card is not used, the number of differentiations is assumed to be zero.

DIFFERENTIATE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	DIFFERENTIATE (D)	8A10	Keyword indicating that zeros are to be added to the orgin of the transfer function.

DIFFERENTIATE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NDIFFR	I5	Number of differentiations.

FORM RATIO OF POLYNOMIALS Card

This control card causes formation of a transfer function from the poles and zeros which have been previously formed by the **COMPUTE POLES** and **COMPUTE ZEROS** control cards. The **FORM RATIO OF POLYNOMIALS** card must be used for establishing a transfer function even if cancellation is not required. Options specified under the **CANCEL**, **DC GAIN**, **INTEGRATE**, and **DIFFERENTIATE** control cards are exercised at this time.

FORM RATIO OF POLYNOMIALS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	FORM RAIO OF POLYNOMIALS (FROP)	8A10	Keyword indicating that a transfer function is to be formed from from the poles and zeros calculated previously.

6.3.5 TIME RESPONSE

Time response calculations are performed by the inverse Laplace method. The inverse Laplace calculations in the QR program permits multiple poles (maximum = 6). Residues are computed according to the transfer function discussed in the Transfer Function Formation section, 6.3.4. Time response computations are obtained from transfer functions which are generated by the QR algorithm. The transfer functions are not obtained directly from the matrix polynomial. The time response is determined at discrete time points selected by the user. Plotting of the resulting response is available in printer-plotter plots form.

• TIME INTERVALS	This card permits time points to be specified for the evaluation of the **TIME RESPONSE**.
• COMPUTE TIME RESPONSE	This card causes a time response to be formed.
• COMPUTE TIME RESPONSE WITH TAU	This card has the same effect as the **COMPUTE TIME RESPONSE** except the forcing function can be turned off after a specified time interval.
• COMPUTE CONVOLUTE	This card permits the user to calculate the time response of any linear system given nearly any input function.
• FORCING FUNCTION	This card permits the user to establish a forcing function that is used with the **COMPUTE CONVOLUTE** card.
• TIME RESPONSE	This card permits the user to control the residue and time response printout.
• PLOT TIME RESPONSE	This card permits the user to specify the plot options of the **TIME RESPONSE**.

TIME INTERVAL Cards

This control card permits time points to be specified for the evaluation of the time response. Time points are listed by means of intervals. Each interval has a starting time, a time increment, and a final time.

TIME INTERVAL card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TIME INTERVALS (TI)	8A10	Keyword indicating that data defining the time intervals is to be read from cards.

TIME INTERVALS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTIME	I5	Number of time intervals (maximum = 6).

TIME INTERVALS card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	TIMEI	E12.5	Starting time for interval.
13-24	TIMED	E12.5	Time increment for interval.
25-36	TIMEF	E12.5	Final time for inerval.

Repeat card-3 as many times as necessary to list all intervals as indicated by NTIME on card-2.

COMPUTE TIME RESPONSE Card

This control card causes a time response to be formed. The poles and zeros established by the FORM RATIO OF POLYNOMIALS control card are used to form residues. The residues are then used to find the time response by inverse Laplace transform techniques. The difference limits established by the CANCEL control card are used to determine if poles are repeated, i.e., close enough to be considered equal. The time response is calculated for the time points specified by the TIME INTERVALS control card.

COMPUTE TIME RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE TIME RESPONSE (CTR)	8A10	Keyword indicating that a time response is to be calculated.

COMPUTE TIME RESPONSE WITH TAU Cards

This control card has the same effect as the **COMPUTE TIME RESPONSE** control card except the forcing function can be turned off after a specified time interval.

COMPUTE TIME RESPONSE WITH TAU card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE TIME RESPONSE WITH TAU (CTRWT)	8A10	Keyword indicating that a time response is to be calculated for a forcing function stopping at time, τ.

COMPUTE TIME RESPONSE WITH TAU card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	TAU	E12.5	Time constant.

COMPUTE CONVOLUTE Card

This control card computes a time history by evaluating the convolution integral shown in section 4.8. Care should be used with the input of f(t) because the function is set to zero when time advances beyond the last time value at which the forcing function is defined. Trapezoidal integration is used by the QR program with the step size being determined by the **TIME INTERVALS** control card. The poles and zeros established by the **FORM RATIO OF POLYNOMIALS** card determines the impulse response, h(t).

COMPUTE CONVOLUTE card-1

COLS.	KEYWORD/ VARIABLES	FORMAT	DESCRIPTION
1-80	COMPUTE CONVOLUTE (CC)	8A10	Keyword indicating that a time response will be calculated using the convolution ingetral.

FORCING FUNCTION Cards

This control card permits the user to establish a forcing function that is used with the COMPUTE CONVOLUTE control card.

FORCING FUNCTION card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	FORCING FUNCTION (FF)	8A10	Keyword indicating that data defining the forcing function to be used in the convolution calculation is to be read from cards.

FORCING FUNCTION card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NFF	I5	The number of data points which define the forcing function. (default=12)

FORCING FUNCTION card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	TIMEI(1)	E12.5	1st TIME value.
13-24	TIMEI(2)	E12.5	2nd TIME value
25-36	TIMEI(3)	E12.5	3rd TIME value.
37-48	TIMEI(4)	E12.5	4th TIME value.
49-60	TIMEI(5)	E12.5	5th TIME value.
61-72	TIMEI(6)	E12.5	6th TIME value.

Repeat card-3 as many times as necessary to list all time values as indicated by NFF on card-2. Omit card-3 if NFF = 0 on card-2.

FORCING FUNCTION card-4

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	FFT(1)	E12.5	1st forcing function value.
13-24	FFT(2)	E12.5	2nd forcing function value.
25-36	FFT(3)	E12.5	3rd forcing function value.
37-48	FFT(4)	E12.5	4th forcing function value.
49-60	FFT(5)	E12.5	5th forcing function value.
61-72	FFT(6)	E12.5	6th forcing function value.

Repeat card-4 as many times as necessary to list all forcing function values. Omit card-4 if NFF = 0 on card-2. It is recommended that the number of data points be kept to a minimum.

TIME RESPONSE Cards

This control card permits the user to control the residue and time response printout.

TIME RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TIME RESPONSE (TR)	8A10	Keyword indicating results of the time response calculation are to be printed according to the specified options.

TIME RESPONSE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	ITIME3	I5	If ITIME3 = 1, print both residues and response (Default). If ITIME3 = 2, print the residues only. If ITIME3 = 3, print the response only. If ITIME3 = 4, print neither residues nor response.

PLOT TIME RESPONSE Cards

This control card permits the user to specify the options desired for plotting of the time response.

PLOT TIME RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PLOT TIME RESPONSE {PTR}	8A10	Keyword indicating that the results of the time response calculation are to be plotted according to the specifications read from cards.

PLOT TIME RESPONSE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	ITIME1	I5	If ITIME1 = 0, no plots are made (Default) If ITIME1 = 1, time response is plotted. (scales are set automatically)
6-10	ITIME2	I5	If ITIME2 = 2, printer-plotter plots only.
11-15	ITIME3	I5	If ITIME3 = 0, no ordinate title is read. If ITIME3 = 1, the title of the ordinate is specified on card-3.
16-20	ITIME4	I5	If ITIME4 = 0, no ordinate scales are read. If ITIME4 = 1, the ordinate scales are specified on card 4.

PLOT TIME RESPONSE card-3

If ITIME 3 = 0 (card-2) omit this card.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-40	TITLE	4A10	Ordinate title.

PLOT TIME RESPONSE card-4

If ITIME 4 = 0 (card-2) omit this card.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	YQTOP(1)	E12.5	Maximum value of ordinate scale.
13-24	YQTOP(2)	E12.5	Minimum value of ordinate scale.

6.3.6 FREQUENCY RESPONSE

Frequency response computations are obtained from the transfer functions which are generated by the QR algorithm using the control cards listed in section 6.3.4. Frequency response calculations are performed by evaluating the transfer function at discrete values of $s = j\omega$.

• COMPUTE FREQUENCY RESPONSE	This card causes a frequency response to be formed.
• GUST SPECTRUM	This card permits input of a gust power spectrum.
• COMPUTE PSD	This card causes a power spectral density (PSD) response to be formed.
• FREQUENCY RESPONSE	This card permits the user to control the printout of the frequency response.
• PLOT FREQUENCY RESPONSE	This card permits the user to specify the plot options of the frequency response.
• PLOT PSD	This card permits specification of the power spectral density plots.

COMPUTE FREQUENCY RESPONSE Card

This control card causes a frequency response to be formed. The poles and zeros established by the FORM RATIO OF POLYNOMIALS control card are used to compute the frequency response. The response is evaluated at frequencies automatically selected by the program. Starting and stopping frequencies are determined by examination of the poles and zeros. Intermediate frequencies are selected by holding the gain change to 2 dB and the phase change to 5 degrees between response points adjacent on the frequency scale. The program computes the following: the minimum gain values, the maximum gain values, the gain margins, and the phase margins. For the above parameters, the direction of travel (increasing or decreasing gain, and clockwise or counter-clockwise phase angle) is printed. The mean square value, RMS value, and characteristic frequency of the response are calculated and printed. These data are not computed if the transfer function has a pole at the origin.

COMPUTE FREQUENCY RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE FREQUENCY RESPONSE {CFR}	8A10	Keyword indicating a frequency response calculation is to be performed.

GUST SPECTRUM Cards

The GUST SPECTRUM control card should be used prior to use of a COMPUTE PSD control card (unless only the response for white noise is desired).

GUST SPECTRUM card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	GUST SPECTRUM {GS}	8A10	Keyword indicating the gust spectrum parameters are to be read.

GUST SPECTRUM card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GAIN(1)	E12.5	σ_x RMS turbulence level of gust (meter/sec) or (ft/sec).
13-24	GAIN(2)	E12.5	L characteristic length (meters) or (ft).
25-36	GAIN(3)	E12.5	V free stream velocity of airplane (meter/sec) or (ft/sec).
37-48	GAIN(4)	E12.5	x coefficients required to create
49-60	GAIN(5)	E12.5	y a von Karmen or Dryden or any other
61-72	GAIN(6)	E12.5	K_1 gust power spectrum.

GUST SPECTRUM card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GAIN(7)	E12.5	K_2 coefficients required to create
13-24	GAIN(8)	E12.5	K_3 a von Karmen or Dryden or any other gust spectrum.

COMPUTE PSD Card

This control card causes a PSD of a response to be formed. A gust power spectrum must first be input by use of a **GUST SPECTRUM** control card. (If the gust spectrum is not specified, white noise is assumed.) The poles and zeros established by the **FORM RATIO OF POLYNOMIALS** control card are used to compute the frequency response. The PSD of the response is evaluated at frequencies automatically selected by the program using the same criteria listed under the **COMPUTE FREQUENCY RESPONSE** control cards. The printing of the summary data and the detailed response is controlled by the **FREQUENCY RESPONSE** control card. Plotting of the results is done according to the specifications of the **PLOT FREQUENCY RESPONSE** control card. Nyquist plots are not formed for the PSD response even if Nyquist plots are specified by **PLOT FREQUENCY RESPONSE** control card.

COMPUTE PSD card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE PSD {CPSD}	8A10	Keyword indicating the power spectral density of a response is to be calculated.

FREQUENCY RESPONSE Cards

This control card permits the user to control the printout of the frequency response or the power spectral density of the response and associated summary data. This card also controls the printout at the frequency response which is automatically computed in the root locus sequence.

FREQUENCY RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	FREQUENCY RESPONSE {FR}	8A10	Keyword indicating that the results of the frequency response calculation are to be printed as specified.

FREQUENCY RESPONSE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	IFRQ1	I5	If IFRQ1 = 0, do not compute frequency response automatically in root locus sequence. If IFRQ1 = 1, compute frequency response automatically in root locus sequence.
6-10	IFRQ2	I5	If IFRQ2 = 1, print both summary and detailed responses. If IFRQ2 = 2, print only summary response (Default). If IFRQ2 = 3, print only detailed response. If IFRQ2 = 4, print neither summary nor detailed responses.

PLOT FREQUENCY RESPONSE Cards

This control card permits the user to specify the plot options of the frequency response. Both Bode and Nyquist plots are available. The user has control over the limits of the amplitude and frequency scale.

PLOT FREQUENCY RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PLOT FREQUENCY RESPONSE (PFR)	8A10	Keyword indicating the the frequency response results are to be plotted as specified.

PLOT FREQUENCY RESPONSE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NPLOTF	I5	Number of regions of frequency scale desired to be plotted. A zero will suppress the plot option. (default)

PLOT FREQUENCY RESPONSE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	IOMGAL(I)	I5	Log to base 10 of minimum frequency. (Rad/sec).
6-10	IOMGAR(I)	I5	Log to base 10 of maximum frequency. (Rad/sec).
11-15	IDBTOP(I)	I5	Maximum value of amplitude scale in dB.
16-20	IBDNYQ(I)	I5	If IBDNYQ(I) = 0, do both Bode and Nyquist plots. If IBDNYQ(I) = 1, do Bode plot only. If IBDNYQ(I) = 2, do Nyquist plot only.
21-25	IDBSCL(I)	I5	If IDBSCL(I) = 0, use the specified limits, IDBTOP(I), for the amplitude scale. If IDBSCL(I) = 1, scale the amplitude automatically.

Repeat card-3 as many times as necessary to read the numbers of regions as indicated by NPLOTF in card-2. Omit card-3 if NPLOTF = 0 on card-2.

PLOT PSD Cards

This control card permits specification of the PSD plots. PSD can be plotted against Hertz or radians/second.

PLOT PSD card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PLOT PSD (PPSD)	8A10	Keyword indicating that the power spectral density results are to be plotted as specified.

PLOT PSD card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NPSPP	I5	Number of areas of frequency scale to plot.
6-10	IDUMMY	I5	If IDUMMY = 0, no ordinate title is to be added.
11-15	IPSD1	I5	If IPSD1 = 1, the title of the ordinate is specified in card-4.
16-20	IPSD2	I5	If IPSD2 = 0, frequencies are in Hertz. If IPSD2 = 1, frequencies are in Radians/second.

PLOT PSD card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	FLR(I,1)	E12.5	Left limit of frequency scale.
13-24	FLR(I,2)	E12.5	Right limit of frequency scale.

Repeat card-3 as many times as necessary to specify plotting of all the frequency areas as indicated by NPSPP on card-2. Omit card-3 if the NSPP = 0 on card-2.

PLOT PSD card-4

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TITLE	8A10	dinate title.

Card-4 is read if IPSD1 = 1 on card-2. Card 4 is omitted if NPSPP = 0.

6.3.7 UNSTEADY AERODYNAMICS OPTIONS

This set of control cards performs root locus, flutter, and frequency response calculations using frequency dependent aerodynamics. The equations are assembled in either one of two forms as explained in section 4.9. In both cases, the matrix polynomial input to QR is assumed to contain the non-frequency dependent matrices. For the unsteady frequency response calculations, one to three independent forcing functions can be used. These vectors are stored in the vector polynomial (see VECTOR control card set). The input vectors are all considered coefficients of s ** 0.

The control cards UNSTEADY AIRFORCE COEFFICIENTS, COMPUTE FLUTTER 2, and COMPUTE UNSTEADY FREQUENCY RESPONSE are used with the form 1 (equation 4.8). The AIRFORCE COEFFICIENTS, COMPUTE FLUTTER and COMPUTE FLUTTER AND ROOT LOCUS are used with form 2 of the equations of motion (equation 4.9).

• AIRFORCE COEFFICIENTS COEFFICIENTS	This card permits the airforce coefficients to be input compatible to form 2 of the equations of motion.
• UNSTEADY AIRFORCE COEFFICIENTS	This card permits the airforce coefficients to input compatible to form 1 of the equations of motions.
• PLOT FLUTTER	This card permits the user to input plotting specification for the COMPUTE FLUTTER options.
• COMPUTE FLUTTER	This card causes the computer to enter the flutter sequence for equations of motion in form 2 format.

• **COMPUTE FLUTTER AND ROOT LOCUS**	This card causes the computer to enter the flutter root locus sequence for equations of motion in form 2 format.
• **COMPUTE FLUTTER 2**	This card causes the computer to enter the flutter sequence for equations of motion in form 1 format.
• **COMPUTE UNSTEADY FREQUENCY RESPONSE**	This card causes the computer to enter the unsteady aerodynamics frequency response calculations for equations of motion form 1 format.

AIRFORCE COEFFICIENTS Cards

This control card permits the airforce coefficients to be input for the COMPUTE FLUTTER and COMPUTE FLUTTER AND ROOT LOCUS options of QR. The aerodynamic forces are assumed to be in a form compatible with form 2 of the equations of motion.

AIRFORCE COEFFICIENTS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	AIRFORCE COEFFICIENTS (AC)	8A10	Keyword indicating that airforce coefficients being input for use with the COMPUTE FLUTTER and the COMPUT FLUTTER AND ROOT LOCUS options.

AIRFORCE COEFFICIENTS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NIGFAA	I5	The number of inverse gamma function as well as the number of altitudes. (maximum of 6)

AIRFORCE COEFFICIENTS card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GINVRS(1)	E12.5	1st inverse gamma value.
13-24	GINVRS(2)	E12.5	2nd inverse gamma value.
25-36	GINVRS(3)	E12.5	3rd inverse gamma value.
37-48	GINVRS(4)	E12.5	4th inverse gamma value.
49-60	GINVRS(5)	E12.5	5th inverse gamma value.
61-72	GINVRS(6)	E12.5	6th inverse gamma value.

AIRFORCE COEFFICIENTS card-4

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	ALT(1)	E12.5	1st altitude in feet (meters)
13-24	ALT(2)	E12.5	2nd altitude in feet (meters)
25-36	ALT(3)	E12.5	3rd altitude in feet (meters)
37-48	ALT(4)	E12.5	4th altitude in feet (meters)
49-60	ALT(5)	E12.5	5th altitude in feet (meters)
61-72	ALT(6)	E12.5	6th altitude in feet (meters)

AIRFORCE COEFFICIENTS card-5

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	RFRL	E12.5	Reduced frequency reference length in meters (inches).

UNSTEADY AIRFORCE COEFFICIENTS Cards

This card permits the airforce coefficients to be input for the COMPUTE FLUTTER 2 and COMPUTE UNSTEADY FREQUENCY RESPONSE options in QR. The aerodynamic forces are assumed to be in a form compatible with form 1 of the equations of motion.

UNSTEADY AIRFORCE COEFFICIENTS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	UNSTEADY AIRFORCE COEFFICIENTS (UAC)	8A10	Keyword indicating that airforce ceofficients being input for use with the COMPUTE FLUTTER 2 and the COMPUTE UNSTEADY FREQUENCY RESPONSE options.

UNSTEADY AIRFORCE COEFFICIENTS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	MACH NO.	E12.5	Mach number.
13-24	RFRL	E12.5	Reduced frequency reference length in inches (meters).
25-36	CF	E12.5	Correction factor.
37-48	RHO	E12.5	Density in kg-sec^2/meters^4 (lb-sec^2/ft^4).
49-60	V	E12.5	Free stream velocity in meter/sec (ft/sec).

UNSTEADY AIRFORCE COEFFICIENTS card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NALT	I5	The number of altitudes at which responses are to be computed for the specified Mach number.

UNSTEADY AIRFORCE COEFFICIENTS card-4

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	ALT(1)	E12.5	1st altitude in feet (meters)
13-24	ALT(2)	E12.5	2nd altitude in feet (meters)
25-36	ALT(3)	E12.5	3rd altitude in feet (meters)
37-48	ALT(4)	E12.5	4th altitude in feet (meters)
49-60	ALT(5)	E12.5	5th altitude in feet (meters)
61-72	ALT(6)	E12.5	6th altitude in feet (meters)

The freestream velocity (true airspeed) and density are automatically calculated at each altitude for standard atmospheric conditions. These computed values are ignored if the velocity and density are specified. This latter option could be used for the case of a non-standard atmosphere or a fluid medium other than air. The program converts input values to in.-lb.-sec., and assumes a consistent set of these units in the analysis.

PLOT FLUTTER Cards

This control card permits the user to input plotting specifications for the COMPUTE FLUTTER options of QR.

PLOT FLUTTER card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PLOT FLUTTER (PF)	8A10	Keyword indicating that the results of the flutter calculation are to be plotted as specified.

PLOT FLUTTER card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NPLTFL	I5	If NPLTFL = 0, no plots. (default) If NPLTFL = 1, printer-plots.

PLOT FLUTTER card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	GMIN	E12.5	Minimum damping value (g).
13-24	GMAX	E12.5	Maximum damping value (g)
25-36	VMIN	E12.5	Minimum velocity in knots (kilometers/hour).
37-48	VMAX	E12.5	Maximum velocity in knots (kilometers/hour).
49-60	CMIN	E12.5	Minimum frequency (Hertz).
61-72	CMAX	E12.5	Maximum frequency (Hertz).

Omit card-3 if NPLTFL = 0 on card-2.

COMPUTE FLUTTER Cards

This control card causes the computer to enter the flutter sequence for equations of motion on the form 2 format. Disk files reserved when this option is used in QR are TAPE5, TAPE6, TAPE7, and TAPE10.

COMPUTE FLUTTER card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE FLUTTER (CF)	8A10	Keyword indicating that flutter calculations will be performed using the equations of motion in form 2.

COMPUTE FLUTTER card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit containing airforce matrices. If negative, the tape will be rewound before any file or matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	NK	I5	The number of airforce matrices given for each value of inverse gamma/altitude specified with AIRFORCE COEFFICIENTS control card.
21-25	NEOFK	I5	If NEOFK = 0, print matrices read from tape. If NEOFK = 1, do not print matrices.

Each matrix on NTAPE is a real matrix of size M by 2M. There will be a total of N matrices on this tape where N is the number of reduced frequencies. Each row of the aerodynamics matri matrix has 2M columns because columns are stored real, imaginary, real, imaginary, etc. The N matrices will be read for each inverse gamma/altitude loop of the flutter option.

CONTENTS

Page

1.0 SUMMARY 1 1/A9
2.0 INTRODUCTION 3 1/A11
3.0 SYMBOLS AND ABBREVIATIONS 4 1/A12
4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION 8 1/B2
  4.1 The Basic Polynomial Equation 8 1/B2
  4.2 Open-Loop and Closed-Loop Systems 10 1/B4
  4.3 Transfer Function Formation 13 1/B7
  4.4 Eigenvalue Calculation 15 1/B9
  4.5 State Space Formulation 16 1/B10
  4.6 Frequency Response Calculation 17 1/B11
  4.7 Power Spectral Density Analysis 18 1/B12
  4.8 Time Response Calculations 21 1/C1
    4.8.1 Time Response by Method of Residues 21 1/C1
    4.8.2 Time Response With Tau 23 1/C3
    4.8.3 Time Response for Forcing Functions that Do Not Have Simple Laplace Transforms 23 1/C3
  4.9 Unsteady Aerodynamic Options 24 1/C4
    4.9.1 Frequency Response Analysis 25 1/C5
    4.9.2 Flutter Analysis 27 1/C7
5.0 PROGRAM STRUCTURE AND DESCRIPTION 29 1/C9
  5.1 QR Program Control and Eigenvalue Rooting 29 1/C9
  5.2 Time Response Calculations 29 1/C9
  5.3 Frequency Response and PSD Calculations 31 1/C12
  5.4 Root Locus and V-g Plotting 31 1/C12
  5.5 Unsteady Airforce Frequency Response and Flutter Calculations 31 1/C12
6.0 COMPUTER PROGRAM USAGE 32 1/C13
  6.1 Control Cards 32 1/C13
  6.2 Resource Estimates 33 1/C13
  6.3 Card Input Data 34 1/D1
    6.3.1 Case Initialization 35 1/D2
    6.3.2 Matrix Element Operations 36 1/D3
    6.3.3 Roots of Matrix and Root Locus 57 1/E10
    6.3.4 Transfer Function Formation 69 1/F8
    6.3.5 Time Response 73 1/F12
    6.3.6 Frequency Response 79 1/G4
    6.3.7 Unsteady Aerodynamics Options 84 1/G9
    6.3.8 Cycling Control Cards and Labeling 94 2/A7
    6.3.9 Input Card Summary 98 2/A11

CONTENTS (CONCLUDED)

Page

6.4 Examples of Required Card Sets for Some Typical Problems 107 2/C1
6.5 Output Data 109 2/C3
6.5.1 Printed Output Data 109 2/C3
6.5.2 File Output 109 2/C3
6.6 Restructions 115 2/C9
6.7 Diagnostics 115 2/C9
6.7.1 READTP Error Codes 118 2/C12
6.7.2 WRTETP Error Codes 119 2/C13
6.8 Use of User Supplies Subroutines 119 2/C13
7.0 SAMPLE PROBLEMS 127 2/D7
7.1 Eigenvalue Calculation 127 2/D7
7.2 Time Response Calculation 128 2/D8
7.3 Frequency Response and Power Spectral Density Calculation 131 2/D11
7.4 Root Locus Calculation 133 2/D13
7.5 Sample Problem Output 135 2/E1
Appendix A: QR Eigenvalue Calculation Process 147 2/F9
Appendix B: QR State Space Formulation 161 2/G9
REFERENCES 175 3/A11

COMPUTE FLUTTER AND ROOT LOCUS Cards

This control card causes the computer to enter the flutter root locus sequence for the equations of motion in form 2 format.

COMPUTE FLUTTER AND ROOT LOCUS card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE FLUTTER AND ROOT LOCUS {CFARL}	8A10	Keyword indicating that flutter and root locus calculations will be performed using the equations of motion in form 2.

COMPUTE FLUTTER AND ROOT LOCUS card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit containing airforce matrices. If negative, the tape will be rewound before any file or matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	NK	I5	The number of airforce matrices given for each value of inverse gamma/altitude specified with AIRFORCE COEFFICIENTS control card.
21-25	NEOFK	I5	If NEOFK = 0, print matrices read from tape. If NEOFK = 1, do not print matrices.

COMPUTE FLUTTER 2 Cards

This control card causes the computer to enter the flutter sequence for the equations of motion in the form 1 format. Disk files reserved when this option is used in QR are TAPE5, TAPE6, TAPE7 and TAPE10. The following data cards are read after the QR program encounters a COMPUTE FLUTTER control card.

COMPUTE FLUTTER 2 card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE FLUTTER 2 {CF2}	8A10	Keyword indicating that flutter calculations will be performed using the equations of motion in form 1.

COMPUTE FLUTTER 2 card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit containing airforce matrices. If negative, the tape will be rewound before any file or matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace that many files.
11-15	NMATR	I5	Number of matrices on tape in sequence of increasing k values. The k values of interest must lie within the range of the lowest and highest k values on tape.
16-20	NK	I5	Number of k value matrices (maximum=30) to be used for each altitude specified in the UNSTEADY AIRFORCE COEFFICIENTS card. The k values listed on card 2 must be in sequence of increasing magnitude. Values of k matrices between those listed on tape will be computed by linear interpolation.
21-25	NEOFK	I5	If NEOFK = 0, [E(k)] on tape will be used in existing form. If NEOFK = 1, [E(k)] on tape has been normalized by k^2; the program will automatically multiply each $\left[E(k)\right]$ by its k^2 value before performing a V-g analysis.
26-30	NPRT	I5	If NPRT = 0, print the [E(k)] matrix used in the analysis. If NPRT = 1, do not print the [E(k)] matrix.

COMPUTE FLUTTER 2 card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-12	KVALUE(1)	E12.5	(List of reduced frequencies for flutter analysis)
13-24	KVALUE(2)	E12.5	
25-36	KVALUE(3)	E12.5	
37-48	KVALUE(4)	E12.5	
49-60	KVALUE(5)	E12.5	
61-72	KVALUE(6)	E12.5	

Card-3 is repeated until NK (card-2) reduced frequencies have been defined.

COMPUTE UNSTEADY FREQUENCY RESPONSE Cards

This control card causes the program to calculate the unsteady aerodynamics frequency response using the equations of motion in form 1 format.

COMPUTE UNSTEADY FREQUENCY RESPONSE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	COMPUTE UNSTEADY FREQUENCY RESPONSE { CUFR }	8A10	Keyword indicating that a frequency response calculation will be performed using the equations of motion in form 1.

COMPUTE UNSTEADY FREQUENCY RESPONSE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NTAPE	I5	Number of tape unit containing airforce matrices. If negative, the tape will be rewound before any file or matrix skipping occurs.
6-10	NFILE	I5	Number of end-of-files to skip past. A minus number will backspace the tape that many files.
11-15	NMAT	I5	Number of matrices to skip past on tape. This number must be positive.
16-20	NK	I5	Number of matrices on tape to be used for each altitude response computation.
21-25	NCLOL	I5	If NCLOL = 0, compute a closed loop response. If NCLOL = 1, compute an open loop response. If NCLOL = 2, a PSD and open loop response will be calculated.
26-30	NFRLIM	I5	If NFRLIM = 0, increment the frequency within the limits dB=4 and ϕ=10. If NFRLIM = 1, increment the frequency within the limits dB=2 and ϕ=5. If NFRLIM = 2, increment the frequency within the limits dB=6 and ϕ=15.
31-35	NEOFK	I5	If NEOFK = 0, $[F(k)]$ on tape will be used in existing form. If NEOFK = 1, $[E(k)]$ on tape has been normalized by k^2: the program will automatically multiply each $[F(k)]$ by its k^2 value before computing a response. The user is responsible for correcting the dimensions to conform to the OR equations of motion format.
36-40	NXMYI	I5	Number of (x_m, y_i) combinations for closed loop response analysis.

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Input additional cards of the same format as card-2 for other (x_m, y_i) input/output combinations.

COMPUTE UNSTEADY FREQUENCY RESPONSE card-3

If NCOL = 1 or 2 (card-2) omit this card.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	NUKNOW	I5	The number of "m" of the output coordinate x_m where "m" lies in the range $1 \leq m \leq n$ corresponding to the n x 2n characteristic matrix.
6-10	NRHSV	I5	The number "i" of the right hand side column vector (e.g., i=1 for a single input problem); $1 \leq i \leq 3$.
11-15	NUPSD	I5	If NUPSD = 0, do not compute a PSD for this (x_m, y_i) combination. If NUPSD = 1, compute a PSD for this (x_m, y_i) combination.

The unsteady frequency response may be calculated for one to three different input vectors. For each input vector for which the frequency response is desired, card-2 and card-3 are repeated.

6.3.8 CYCLING CONTROL CARDS AND LABELING

This set of QR control cards are used to define a block of data which may be repeated in the data input flow, to re-title the case being processed, to define plot titles, and to terminate execution.

• SEQUENCE	The SEQUENCE card permits the user to assign an arbitrary name to a set of sequential data cards.
• TITLE	This card permits the case title to be changed.
• PTITLE	This card permits titles to be written in the Boeing title block (for use only on Boeing computers)
• IDPLOT	This card permits plot identification information to be read and must appear as the first card in the data deck.
• STOP	Use this card defines the end of the data stack (not required).

SEQUENCE Cards

Sets of control cards tend to appear often in a single computer run. The SEQUENCE control control card set permits the user to assign an arbitrary name to a set of sequential data cards. Subsequent references to that set of cards can be made by inserting a data card with the set name into the card stack. Thus, the appearance of the name causes the same effect as if the set of data cards had been inserted in the card stack. (For use on Boeing computer only).

SEQUENCE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-9	SEQUENCE	A10	Keyword indicating a repeatable block of input data is to be defined.
10-15	XXXXXX	A6	A 1 to 6 character name assigned to the data block. Alphanumeric, special symbols and blanks may be used for the name.

A typical set of cards is in the following example:

```
SEQUENCE $LOOP2
CRAMER REPLACE
        10
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
COMPUTE FREQUENCY RESPONSE
INTEGRATE
         7
FORM RATIO OF POLYNOMIALS
COMPUTE TIME RESPONSE
INTEGRATE
         0
REPLACE
END
```

Once the above sequence has been established, appearance of $LOOP2 (starting in card column 1) will have the same effect as the 12 data cards after the SEQUENCE $LOOP2 card and before the END card.

SEQUENCE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	END	8A10	Keyword indicating the end of a repeatable data block.

TITLE Cards

This control card permits the case title to be changed. The title is originally entered on BEGIN card-2 in the sequence initiated by the BEGIN control card. The case title is printed with the root locus output and appears on all plots.

TITLE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TITLE { T }	8A10	Keyword indicating a new job title will be read and used.

TITLE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	TITLE	8A10	TITLE (The available symbols are: 0-9, A-Z, = + - / · , ( ) $ *)

PTITLE Cards

This control card permits titles to be written in the Boeing title block. (For use only on Boeing computers.)

PTITLE card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	PTITLE { PT }	8A10	Keyword indicating a set of plot titles will be read.

PTITLE card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	IEND	I5	Numbers of titles to put into the title block (maximum=3). If IEND = 0, plot title will be supressed.

PTITLE card-3

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-40	KTITLE	4A10	TITLE (The available symbols are: 0-9, A-Z, = + - / . , ( ) $ *)

Repeat card-3 as many times as necessary to list the specified number of titles as indicated by IEND (card-2). Omit card-3 if IEND (card-2) is zero.

IDPLOT Cards

This control card defines user identification information for any plotting to be done. The card set must appear as the first card set in the data deck. Use this card set only once per computer run.

IDPLOT card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	IDPLOT	8A10	Keyword indicating plot identification data is being input.

IDPLOT card-2

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-30	ROW(1)	3A10	User's name.
31-40	ROW(4)	A10	Log station.
41-50	ROW(5)	A10	Phone number.

STOP Card

Use this card only at the end of the data stack (optional). This card causes an end-of-file mark to be written on TAPE99.

STOP card-1

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-80	STOP	8A10	Keyword indicating the end of the data stack.

One end-of-file mark is written onto TAPE99.

Requirements or Function	Key Words and/or Variables	Card Format	Reference Card Set
PROBLEM INITIALIZATION			
If plotting is to be done, IDPLOT card set should appear before BEGIN card set.	BEGIN (B)	8A10	card-1
	Title(optional)	8A10	card-2
	MATORD MATDE NGR NODIAG UNITS	5I5	card-3
INPUT OF MATRIX AND VECTOR POLYNOMIALS			
Mat. polyn. read from cards. End input with blank card.	MATRIX (M)	8A10	card-1
	II JJ KK U(II,JJ,KK)	4(3I2, E14.7)	card-2
Complex mat. polyn. read from cards. End input with blank card.	MATRIX COMPLEX (MC)	8A10	card-1
	II JJ KK $U(II,JJ,KK)_{Re}$ $U(II,JJ,KK)_{Im}$	2(3I2, E14.7,6X, E14.7)	card-2
Card input for a particular partition of the mat. poly.	CONTINUOUS MATRIX (CON)	8A10	card-1
	MATORD MTDEG	2I5	card-2
	U(1,1,MTDEG) U(1,2,MTDEG)	6E12.5	card-3
Mat. poly. read from tape. End input with blank card.	READ MATRIX TAPE (RMT)	8A10	card-1
	NTAPE NFILES NMAT MATDG NPRT	5I5	card-2
Vector polyn. read from cards. End input with a blank card.	VECTOR (V)	8A10	card-1
	J1 J2 VEC(J1,J2)	4(I2,2X, I2,E14.7)	card-2
Vec. polyn. read from tape. End input with blank card.	READ VECTOR TAPE (RVT)	8A10	card-1
	NTAPE NFILES NMAT MATDG NPRT	5I5	card-2

Requirements or Function	Key Words and/or Variables	Card Format	Reference Card Set
	MATRIX OPERATIONS		
Replace elem. of mat. polyn. End input with blank card.	REPLACE (R)	8A10	card-1
	II JJ KK U(II,JJ,KK)	4(3I2, E14.7)	card-2
Replace elem. of complex mat. polyn. End input with blank card.	REPLACE COMPLEX (RC)	8A10	card-1
	II JJ KK $U(II,JJ,KK)_{Re}$ $U(II,JJ,KK)_{Im}$	2(3I2, E14.7,6X, E14.7)	card-2
Replace elem. of vector polyn. End input with blank card.	REPLACE VECTOR (RV)	8A10	card-1
	J1 J2 VEC(J1,J2)	4(I2,2X,I2, E14.7)	card-2
Rep. col. of mat. polyn. with vec. polyn.	CRAMER REPLACE (CR)	8A10	card-1
	J L	2I5	card-2
Elim. dynamic char. of selected modes from mat. poly.	DELETE (DE)	8A10	card-1
	M	I5	card-2
	KFIX1 KFIX2 KFIX3	16I5	card-3
Elim. dynamic char. of selected modes from mat. polyn. and reduce size of mat. polyn.	DELETE AND REDUCE (DAR)	8A10	card-1
	M	I5	card-2
	KFIX1 KFIX2 KFIX3	16I5	card-3
Elim. selected modes from the matrix polyn.	TRUNCATE (TRU)	8A10	card-1
	M	I5	card-2
	KFIX1 KFIX2 KFIX3	16I5	card-3

Requirements or Function	Key Words and/or Variables	Card Format	Reference Card Set
Elim. selected modes from mat. polyn. and reduce the size of the mat. polyn.	TRUNCATE AND REDUCE {TAR}	8A10	card-1
	M	I5	card-2
	KFIX1 KFIX2 KFIX3	16I5	card-3
	ADDITION OF SENSOR EQUATIONS		
Define BS loc. at which incoming sensor equations are defined.	BODY STATIONS {BS}	8A10	card-1
	NBS	I5	card-2
	X(1) X(2) X(3)	6E12.5	card-3
Sensor equations read from tape	SENSOR {SE}	8A10	card-1
	NTAPE NFILE NMAT ICYCLE	4I5	card-2
Desired loc. of sen.	BS	E12.5	card-3
Loc. of sen. in mat. polyn.	IQROW IQPOW ILIM	3I5	card-4
	OUTPUT OF MATRIX POLYNOMIAL		
Print mat. polyn.	PRINT {P}	8A10	card-1
Punch mat. polyn.	PUNCH {PU}	8A10	card-1
Print a portion of the mat. polyn.	PRINT PARTIAL MATRIX {PPM}	8A10	card-1
	I1 I2 I3 I4 I5 I6	6I5	card-2
Magnetic file manipulation.	SPACE TAPE {ST}	8A10	card-1
	NTAPE NFILE NMAT	3I5	card-2

Requirements or Functions	Key Words and/or Variables	Card Format	Reference Card Set
CALCULATION OF MATRIX ROOTS			
Calc. roots of matrix polyn.	COMPUTE {CO}	8A10	card-1
Calc. roots and store as poles.	COMPUTE POLES {CP}	8A10	card-1
Calc. roots and store as zeros.	COMPUTE ZEROS {CZ}	8A10	card-1
Calc. roots and save for plotting.	COMPUTE LOCUS {CL}	8A10	card-1
CALCULATION OF ROOT LOCUS			
Definition of gain magnitudes and phases to be used for root locus calc.	GAINS {G}	8A10	card-1
	NGAIN NPHASE	2I5	card-2
	GAIN(1) GAIN(2) GAIN(3)	6E12.5	card-3
	PHASE(1) PHASE(2) PHASE(3)	6E12.5	card-4
Def. of gain locations	NPLOC	I5	card-5
	IRCM(1) IRCM(2) IRCM(3)	4(3I2, 14X)	card-6
	COMPUTE ROOT LOCUS {CRL}	8A10	card-1
	COMPUTE ROOT LOCUS WITH PREVIOUS POLES {CRLWPP}	8A10	card-1
	COMPUTE ROOT LOCUS WITH PREVIOUS ZEROS {CRLWPZ}	8A10	card-1
	COMPUTE ROOT LOCUS WITH PREVIOUS POLES AND ZEROS {CRLWPPAZ}	8A10	card-1
Charc. polyn. formed for all root calc. which come after this card.	FORM POLYNOMIAL {FP}	8A10	card-1
Cancel FORM POLYNOMIAL card.	NO FORM POLYNOMIAL {NFP}	8A10	card-1

Requirements or Functions	Key Words and/or Variables	Card Format	Reference Card Set
ROOT LOCUS PLOTTING			
Spec. for plotting root locus.	PLOT ROOT LOCUS (PRL)	8A10	card-1
	NPLOT NPHA IRLPF	3I5	card-2
Omit if NPLOT=0.	XLFT XRT YTOP YBOT	4E12.5	card-3
	EXECUTE PLOTS (EP)	8A10	card-1
Root locus and poles(marked with x's) are plotted.	EXECUTE PLOTS WITH POLES (EPWP)	8A10	card-1
Root locus and zeros(marked with o's) are plotted.	EXECUTE PLOTS WITH ZEROS (EPWZ)	8A10	card-1
Root locus,poles (x's) and zeros (o's) are plotted.	EXECUTE PLOTS WITH POLES AND ZEROS (EPWPAZ)	8A10	card-1
STATE VARIABLE FORMATION			
	STATE (STA)	8A10	card-1
TRANSFER FUNCTION FORMATION			
Add poles at origin of T.F.	INTEGRATE (I)	8A10	card-1
	NINTEG	I5	card-2
Add zeros at origin of T.F.	DIFFERENTIATE (D)	8A10	card-1
	NDIFFR	I5	card-2
Eliminate opole/zero pairs	CANCEL (C)	8A10	card-1
	XREAL XIMAG CRATIO	3E12.5	card-2

Requirements or Function	Key Word and/or Variables	Card Format	Reference Card Set
Add scale factors to T.F.	DC GAIN {DCG}	8A10	card-1
	NGAINP NGAINZ	2I5	card-2
Omit if NGAINP=0.	GAINP(1) GAINP(2) GAINP(3)	6E12.5	card-3
Omit if NGAINZ=0.	GAINZ(1) GAINZ(2) GAINZ(3)	6E12.5	card-4
Form T.F.	FORM RATIO OF POLYNOMIALS {FROP}	8A10	card-1
TIME RESPONSE CALCULATIONS			
Def. time point intervals	TIME INTERVALS {TI}	8A10	card-1
	NTIME	I5	card-2
Repeat NTIME times	TIMEI TIMED TIMEF	3E12.5	card-3
	COMPUTE TIME RESPONSE {CTR}	8A10	card-1
Stop forcing func. at t=TAU	COMPUTE TIME RESPONSE WITH TAU {CTRWT}	8A10	card-1
	TAU	E12.5	card-2
Define forcing func. to use in convol.	FORCING FUNCTION {FF}	8A10	card-1
	NFF	I5	card-2
	TIMEI(1) TIMEI(2) TIME(3)	6E12.5	card-3
	FFT(1) FFT(2) FFT(3)	6E12.5	card-4
	COMPUTE CONVOLUTE {CC}	8A10	card-1
TIME RESPONSE OUTPUT			
Cont. time resp. and resid. printout.	TIME RESPONSE {TR}	8A10	card-1
	ITIME3	I5	card-2

Requirements or Function	Key Word and/or Variables	Card Format	Reference Card Set
Plot options	PLOT TIME RESPONSE {PTR}	8A10	card-1
	ITIME1 ITIME2 ITIME3 ITIME4	4I5	card-2
Omit if ITIME3=0.	Title	8A10	card-3
Omit if ITIME4=0	YQTOP(1) YQTOP(2)	2E12.5	card-4
FREQUENCY RESPONSE AND PSD CALCULATIONS			
	COMPUTE FREQUENCY RESPONSE {CFR}	8A10	card-1
Define gust spectrum for PSD calc.	GUST SPECTRUM {GS}	8A10	card-1
	SIGMAX L V X Y K1	6E12.5	card-2
	K2 K3	2E12.5	card-3
	COMPUTE PSD {CPSD}	8A10	card-1
FREQUENCY RESPONSE AND PSD OUTPUT			
Print cont.	FREQUENCY RESPONSE {FR}	8A10	card-1
	IFRQ1 IFRQ2	2I5	card-2
Plot options	PLOT FREQUENCY RESPONSE {PFR}	8A10	card-1
	NPLOTF	I5	card-2
Repeat NPLOTF times Omit if NPLOTF = 0.	IOMGAL(I) IOMGAP(I) IDBTOP(I) IBDNYQ(I) IDBSCL(I)	5I5	card-3
	PLOT PSD {PPSD}	8A10	card-1
	NPSPP IDUMMY IPSD1 IPSD2	4I5	card-2
Repeat NPSPP times Omit if NPSPP=0.	FLR(I,1) FLR(I,2)	2E12.5	card-3
Omit if IPSD1=0 or NPSPP=0.	Tittle	8A10	card-4

Requirements or Function	Key Word and/or Variables	Card Format	Reference Card Set
UNSTEADY AERODYNAMIC COEFFICIENT PARAMETERS			
For form-2 eqs. format.	AIRFORCE COEFFICIENTS (AC)	8A10	card-1
No. of inver. gamma values.	NIGFAA	I5	card-2
Inver. gamma val.	GINVRS(1) GINVRS(2) GINVRS(3)	6E12.5	card-3
Altitude val.	ALT(1) ALT(2) ALT(3)	6E12.5	card-4
Ref. lenght	RFRL	E12.5	card-5
For form-1 eq. format	UNSTEADY AIRFORCE COEFFICIENTS (UAC)	8A10	card-1
Cond. par.	M RFRL CF RHO V	5E12.5	card-2
No. of alt. val.	NALT	I5	card-3
Alt. val.	ALT(1) ALT(2) ALT(3)	6E12.5	card-4
FLUTTER, ROOT LOCUS AND FREQUENCY RESPONSE CACULATIONS			
For form-2 eq. format.	COMPUTE FLUTTER (CF)	8A10	card-1
Mat. read parm.	NTAPE NFILE NMAT NK NEOFK	5I5	card-2
For form-2 eq. format.	COMPUTE FLUTTER AND ROOT LOCUS (CFARL)	8A10	card-1
Mat. read parm.	NTAPE NFILE NMAT NK NEOFK	5I5	card-2
For form-1 eq. format.	COMPUTE FLUTTER 2 (CF2)	8A10	card-1
Mat. read pram.	NTAPE NFILE NMATR NK NEOFK NPRT	6I5	card-2
Red. freq. val.	KVALUE(1) KVALUE(2) KVALUE(3)	6E12.5	card-3

Requirements or Functions	Key Word and/or Variables	Card Format	Reference Card Set
For form-1 eq. format.	COMPUTE UNSTEADY FREQUENCY RESPONSE (CUFR)	8A10	card-1
Mat.read pram. and sol. opt.	NTAPE NFILE NMAT NK NCLOL NFRLIM NEOFK NXMYI	8I5	card-2
	NUKNOW NRHSV NUPSD	3I5	card-3
	FLUTTER SOLUTION PLOTTING		
Plot opt.	PLOT FLUTTER (PF)	8A10	card-1
	NPLTFL	I5	card-2
Omit if NPLTFL=0.	GMIN GMAX VMIN VMAX CMIN CMAX	6E12.5	card-3
	INPUT CYCLING AND LABELING OPTIONS		
Input data block def.	SEQUENCE XXXXXX	A10,A6	card-1
	END	8A10	card-2
Run title change opt.	TITLE (T)	8A10	card-1
	Title	8A10	card-2
Plot title opt.	PTITLE (PT)	4A10	card-1
	IEND	I5	card-2
Repeat IEND times. Omit if IEND=0.	Title	8A10	card-3
If plotting is to be done, IDPLOT c. s. should be placed before BEGIN c.s.	IDPLOT	8A10	card-1
	ROW(1) ROW(2) ROW(3) ROW(4) ROW(5)	5A10	card-2
	STOP (STO)	8A10	card-1

6.4 EXAMPLES OF REQUIRED CARD SETS FOR SOME TYPICAL PROBLEMS

When a card set is listed all associated data cards must be included. In the examples 1 thru 7 the matrix polynomial and vector polynomial are read from cards. In examples 8 and 9, the matrix polynomial is read from magnetic file.

1. Compute the roots of the matrix.

```
BEGIN
MATRIX
COMPUTE
```

2. Compute a root locus.

```
BEGIN
MATRIX
GAINS
COMPUTE ROOT LOCUS
```

3. Transfer function formation.

```
BEGIN
MATRIX
COMPUTE POLES
CRAMER REPLACE
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
```

4. Compute a time response.

```
BEGIN
MATRIX
COMPUTE POLES
CRAMER REPLACE
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
TIME INTERVALS
COMPUTE TIME RESPONSE
```

5. Compute a time response using the convolution integral.

```
BEGIN
MATRIX
COMPUTE POLES
REPLACE
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
FORCING FUNCTION
TIME RESPONSE
PLOT TIME RESPONSE
COMPUTE CONVOLUTE
```

6. Compute frequency response.

```
BEGIN
MATRIX
VECTOR
COMPUTE POLES
CRAMER REPLACE
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
COMPUTE FREQUENCY RESPONSE
```

7. Compute the PSD of a response.

```
BEGIN
MATRIX
COMPUTE POLES
REPLACE
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
GUST SPECTRUM
FREQUENCY RESPONSE
PLOT PSD
COMPUTE PSD
```

8. Perform a flutter analysis using equations in form 2 format.

```
BEGIN
AIRFORCE COEFFICIENTS
READ MATRIX TAPE
PLOT FLUTTER
COMPUTE FLUTTER
```

9. Perform a flutter analysis using equations in form 1 format.

```
BEGIN
READ MATRIX TAPE
PLOT FLUTTER
UNSTEADY AIRFORCE COEFFICIENTS
COMPUTE FLUTTER 2
```

6.5 OUTPUT DATA

6.5.1 PRINTED OUTPUT DATA

The program will initialize and terminate the printed output, with special pages indicating the program name, version, and date of run. An attempt has been made throughout this document to describe outputs generated by the program.

6.5.2 FILE OUTPUT

QR has the capability of creating an output file of plotting vectors. When requested by the user, plotting vectors are generated from each frequency response, power spectral density, time response, root locus, and flutter response solution completed. The plotting vectors are written on TAPE92. The general format for TAPE92 is shown in figure 6. The stacking order of vectors on TAPE92 is dependent upon the solution order used in the QR input data. Note that the general arrangement of plotting vectors consists of an identification array (VIDA), followed by 2 through 6 vector arrays. The number of vector arrays is a function of the type of solution being plotted.

TAPE92 is written in READTP/WRTETP format (ref. 1).

Size of Matrix	Matrix	Description
32 x 1	{VIDA}	Vector identification array
M x 1	{PV1}	First plot vector
M x 1	{PV2}	Second plot vector
M x 1	{PV3}	Third plot vector
M x 1	{PV4}	Fourth plot vector
M x 1	{PV5}	Fifth plot vector
M x 1	{PV6}	Sixth plot vector
	END-OF-FILE	

where:

M = the number of points to be plotted. Matrices VIDA thru PV6 are repeated for each solution for which plotting is requested.

The number of plot vectors present and their contents vary with the type of solution. The minimum number of plot vectors is two and the maximum number is six.

Figure 6. – Contents of TAPE92

Table 1. – TAPE92 Matrix Definition for a Frequency Response Solution

Matrix	Elements	Description
VIDA	VIDA (1) thru VIDA (8)	8 words from BEGIN or TITLE data set
	VIDA (9) thru VIDA (20)	12 words from PTITLE data
	VIDA (21)	Minimum frequency for plot grid
	VIDA (22)	Maximum frequency for plot grid
	VIDA (23)	Minimum dB
	VIDA (24)	Maximum dB
	VIDA (25) thru VIDA (32)	Not used
PV1	1 thru M	Frequency array in radians
PV3	1 thru M	dB value array
PV4	1 thru M	Array of phase angles in degrees
PV5	–	Not present
PV6	–	Not present

The READTP/WRTETP variable NAME (ref 1) is equal to 1 for this solution.

Table 2. – TAPE92 Matrix Definition for a Power Spectral Density Solution

Matrix	Elements	Description
VIDA	VIDA (1) thru VIDA (20)	As defined in table 1
	VIDA (21)	Minimum frequency for plot grid
	VIDA (22)	Maximum frequency for plot grid
	VIDA (23)	Minimum value of PSD of response
	VIDA (24)	Maximum value of PSD of response
	VIDA (25) thru VIDA (32)	PSD plot label
PV1	1 thru M	Frequency in radians
PV2	1 thru M	Response spectrum values
PV3	1 thru M	Percent PSD
PV4	–	Not present
PV5	–	Not present
PV6	–	Not present

The READTP/WRTETP variable NAME (ref 1) is equal to 2 for this solution.

Table 3. – TAPE92 Matrix Definition for a Time History Solution

Matrix	Elements	Description
VIDA	VIDA (1) thru VIDA (20	As defined in table 1
	VIDA (21)	Minimum time for plot grid
	VIDA (22)	Maximum time for plot grid
	VIDA (23)	Minimum time response for plot grid
	VIDA (24)	Maximum time response for plot grid
	VIDA (25) thru VIDA (28)	Ordinate label for plot grid
	VIDA (29) thru VIDA (32)	Not used
PV1	1 thru M	Time values
PV2	1 thru M	Response values
PV3	–	Not present
PV4	–	Not present
PV5	–	Not present
PV6	–	Not present

The READTP/WRTETP variable NAME (ref 1) is equal to 3 for this solution.

Table 4. – TAPE92 Matrix Definition for a Root Locus Solution

Matrix	Elements	Description
VIDA	VIDA (1) thru VIDA (20)	Same as defined in table 1
	VIDA (21)	Minimum real axis for plot grid
	VIDA (22)	Maximum real axis for plot grid
	VIDA (23)	Minimum imaginary axis for plot grid
	VIDA (24)	Maximum imaginary axis for plot grid
	VIDA (25) thru VIDA (32)	Not used
PV1	1 thru M	Real values of loci
PV2	1 thru M	Imaginary values at loci
PV3	1 thru M	Real values of pole(s)
PV4	1 thru M	Imaginary values of pole(s)
PV5	1 thru M	Real values of zero(s)
PV6	1 thru M	Imaginary values of zero(s)

The READTP/WRTETP variable NAME (ref 1) is equal to 4 for this solution.

Table 5. – TAPE92 Matrix Definition for a Flutter Solution

Matrix	Elements	Description
VIDA	VIDA (1) thru VIDA (20)	Same as defined in table 1
	VIDA (21)	Minimum specified velocity in knots (kilometers/hr) for plot grid
	VIDA (22)	Maximum specified velocity in knots (kilometers/hr) for plot grid
	VIDA (23)	Minimum damping value for plot grid
	VIDA (24)	Maximum damping value for plot grid
	VIDA (25)	Minimum Hertz value for plot grid
	VIDA (26)	Maximum Hertz value for plot grid
	VIDA (27) thru VIDA (32)	Not used
PV1	1 thru M	Velocity values in kilometers/hr (knots)
PV2	1 thru M	Damping values
PV3	1 thru M	Hertz values
PV4	–	Not present
PV5	–	Not present
PV6	–	Not present

The READTP/WRTETP variable NAME (ref 1) is equal to 5 for this solution.

6.6 RESTRICTIONS

QR problem size maximums are shown below:

1. The number of eigenvalues for a matrix cannot exceed 140. (Default 60)
2. READTP matrices cannot exceed 70 x 70. (Default 30 x 30)
3. The maximum number of time history values is 1000.
4. The maximum number of repeated poles in the time response calculations is 6.
5. The maximum number of root locus or flutter eigenvalues points for plotting is 2500.
6. The maximum vector size is 120.
7. The maximum gains number of gain magnitudes which can be used to define complex gains is 48.
8. The maximum number of phase values which can be used to define complex gains is 6.
9. The maximum gain locations array size is 8.
10. The maximum number of body stations at which sensor data may be input is 8.
11. The maximum number of data points which can be used to define an arbitrary forcing function is 12.
12. The maximum number of reduced frequencies is 70. (Default 30)

6.7 DIAGNOSTICS

This section contains a listing of all diagnostics that may be printed as a result of errors in input data. In most cases, corrective action taken by the program is explained in the diagnostic. In other cases, no corrective action is taken. In any case, an effort has been made to make the diagnostics as self-explanatory as possible. Only tape positioning errors will cause termination of the execution.

XX's represents data supplied by the program.

The diagnostics are:

1. (THIS IS A COMMENT CARD)
2. THE INPUT MATRIX IS SINGULAR

   Note: When QR detects a singular matrix, the matrix that was being rooted is brought back into core storage, a trace flag is set, and QR reattempts the specified rooting again. Each major step performed by the program is printed onto the output file until the singularity condition is printed (e.g., ROW 1 IS ZERO).

3. ROOT LOCUS IMPOSSIBLE WITHOUT GAINS

4. ASSIST NOT FUNCTIONAL UNDER KTS (KRONOS TIME SHARING)

5. THE ITERATION COUNT FOR ROOT (X.XX, X.XX) IS XX.

6. ELEMENT X REJECTED

   Note: X is 1, 2, 3 or 4 and refers to columns 1-20, 21-40, 41-60 or 61-80 respectively of a matrix elements card.

7. NO BEGIN CARD

8. ILLEGAL MATRIX

   Note: QR has attempted to write the input matrix on a disk file and cannot because the length of it is less than or equal to zero. This generally occurs if the BEGIN card and its respective data is incorrect; however, other program input errors may also cause this comment to be printed.

9. TAPE SPACING ERROR

   Note: Program execution is terminated because of this error.

10. XXXXXXXXXX SEQUENCE NOT FOUND

11. THE NUMBER OF ROWS OF THE INPUT MATRIX DOES NOT EQUAL THE NUMBER OF BODY STATIONS

12. THE MATRIX DIMENSIONS ON TAPE EXCEED THE MATRIX DIMENSIONS OF THE EQUATIONS

13. THE MATRIX DIMENSIONS ON TAPE ARE IN ERROR

14. ELEMENT X X X.XX REJECTED

    Note: A vector element has been incorrectly specified.

15. ZEROS CANNOT BE FOUND

16. THE FIRST CARD IN A QR EXECUTION SHOULD BE AN EXACT DUPLICATION OF YOUR ACCOUNT CARD. THE FOLLOWING WAS USED.

17. THE NUMBER OF POLES MUST BE GREATER/EQUAL TO THE NUMBER OF ZEROS FOR PROPER COMPUTATION OF TIME RESPONSE.

18. NO TIME RESPONSE, MULTIPLE POLES.

19. A MAXIMUM OF 1000 TIME POINTS WILL BE CALCULATED FOR TIME RESPONSE.

20. THE FREQUENCY RESPONSE CANNOT BE DONE WITHOUT POLES OR ZEROS.

21. QR CANNOT PLOT ALL FLUTTER POINTS.

22. QR CANNOT PLOT ALL ROOT LOCUS POINTS.

23. ERRORS IN FREQ. GRID PTS., NPLOTF SET 0.

Note: The Bode/Nyquist plot definitions have been incorrectly entered; the user should check the data cards following the PLOT FREQUENCY RESPONSE command.

24. ERROR IN GUST SPECTRUM, GUST SET 1.

25. THE NUMBER OF GAIN LOCATIONS WAS LE 0, QR IS ASSUMING 1.

26. ERROR IN GAINS, NUMBER GAINS = PHASES = 0.

27. ERROR IN DC GAIN.

28. ERROR IN PLOT ROOT LOCUS, NPLOT = 0.

29. SCRATCH STORAGE ARRAY EXCEEDED, PROGRAM TERMINATING.

Note: The root array in QR has been filled. The user should review his problem for correctness and determine the number of roots he expects.

30. ERROR IN BODY STATION INPUT, NBS = 0.

31. NBS = 0, SENSOR OPTION IMPOSSIBLE.

32. ***WARNING*** XXX XXXX HAVE BEEN SPECIFIED SUBROUTINE BIGGER MUST HAVE BEEN SUPPLIED TO OBTAIN SUCCESSFUL EXECUTION.

Note: This diagnostic will occur for overflow of the following arrays in the QR program: VECTOR, GAIN, FORCING FUNCTION, K VALUE, PHASE, GAIN LOCATIONS, BODY STATIONS, ROOTS and RWBUFF.

33. ERROR READING TAU, NO TIME RESPONSE.

34. THE VALUE OF TAU WAS LESS THAN OR EQUAL 0, TIME RESPONSE WITH NO TIME DELAY FOLLOWS.

35. ERROR IN FREQUENCY MIN, MAX OR INC.

36. FREQUENCY INC. SPECIFIED LESS THAN OR EQUAL 0.

37. ROOT LOCUS PLOT ERROR, XLEFT (XXX.XX) IS GREATER THAN XRIGHT (XXX.XX), PLOT NUMBER XX SKIPPED.

38. ROOT LOCUS PLOT ERROR, YBOT (XXX.XX) IS GREATER THAN YTOP (XXX.XX), PLOT NUMBER XX SKIPPED.

39. THERE MAY BE MORE ROOTS THAN QR CAN STORE WITHOUT SUBROUTINE BIGGER.

6.7.1 READTP ERROR CODES

Error Code	
= 0	If no errors are detected during reading.
= 1000+I	If an FSF (forward space file) error occurred, where I is the number of file marks remaining to be skipped when an end-of-information was encountered.
= 2	If the number of matrices or files to be skipped, before reading starts, is less than zero.
= 3	If the dimensioned number of rows in the matrix is less than or equal to zero.
= 3000+I	If an FSR (forward space record) error occurred, where I is the number of records remaining to be skipped when either an end-of-file or end-of-information was encountered.
= 4	Number of rows in the matrix is greater than the dimensioned row size in the program.
= 5	If the name check failed.
= 6	If the number of rows in the matrix (M) times the number of columns (N) is greater than the buffer size, or M*N=0.
= 7	An end-of-file was read. If it occurs while reading the matrix ID, no information is stored in the user's area. If it occurs while reading the matrix, the ID information will be stored. Note that the records will always be in pairs and an end-of-file should always be encountered with the ID record.

6.7.2 WRTETP ERROR CODES

Error Code	
= 0	If no errors are detected during writing.
= 1000+I	If an FSF (forward space file) error occurred, where I is the number of file marks remaining to be skipped when an end-of-information was encountered.
= 2	If the number of matrices or files to be skipped, before writing starts, is less than zero.
= 3	If the dimensioned number of rows in the matrix is less than or equal to zero.
= 3000+I	If an FSR (forward space record) error occurred, where I is the number of records remaining to be skipped when either an end-of-file or an end-of-information was encountered.
= 4	If the actual number of rows is greater than the dimensioned number of rows in the matrix.
= 6	If the number of rows in the matrix (M) times the number of columns (N) is greater than the buffer size.

6.8 USE OF USER-SUPPLIED SUBROUTINES

There are three additional control cards which QR will recognize. However, the user must supply compiled FORTRAN subroutines in order to execute them correctly. The three control cards are:

Control Card	Action Initiated by QR
CALL MUSER	**Branch to SUBROUTINE MUSER**
CALL VUSER	**Branch to SUBROUTINE VUSER**
CALL USERSB	**Branch to SUBROUTINE USERSB**

SUBROUTINE MUSER

The principal purpose of subroutine MUSER is to define elements of the square polynomial matrix. Thus CALL MUSER performs the same function as MATRIX or REPLACE control cards.

```
SUBROUTINE MUSER (A, M, N, IMAT)
COMPLEX A(M, M, N)
```

where:

A	is a complex matrix whose dimensions are M x M x N.
M	is the row and column dimension of the A matrix.
N	is the highest power of s plus 1.
IMAT	is a count of the number of times the CALL MUSER data card has been used. thus IMAT would be an integer 1 upon the first entry into MUSER.

A sample of a FORTRAN listing for subroutine MUSER is shown in figure 7.

```
      SUBROUTINE MUSER(A,MQ,NQ,IMAT)
      COMPLEX A(MQ,MQ,NQ)
C
C     SET UP NOMENCLATURE TO CONVENIENTLY REFERENCE S0,S1 ETC.MATRICES.
C
      INTEGER S0,S1,S2,S3,S4
      DATA S0,S1,S2,S3,S4/1,2,3,4,5/
      NAMELIST/VALUES/S,B,K,KF,H,VPK,GW,ALPHAG,IXX,IZZ,IXZ,CLB,CLP,CLR,
     1CLDAG,CLDRG,CNB,CNBD,CNP,CNR,CNDAG,CNDRG,CYB,CYP,CYR,CYDAG,CYDRG
C
C     DECLARE ALL REAL VARIABLES BEGINNING WITH I,J,K,L,M,N REAL.
C
      REAL IXZ,IXX,IZZ,M,L1,L2,MACH,K,KF
C
C     ZERO THE A MATRIX UPON FIRST ENTRY.ON 2ND ENTRY,REPLACE ELEMENT
C     11,14,S**2. FOR ENTERIES GREATER THAN 2,THE A MATRIX IS ZEROED,
C     NAMELIST DATA IS READ AND THE ENTIRE A MATRIX IS RECALCULATED.
C     SUBROUTINE CLEAR WILL ZERO AN ARRAY.IN THE CALLING SEQUENCE BELOW
C     THE A MATRIX IS THE ARRAY AND IT IS ZEROED FOR M*M*N*2 ELEMENTS.
C     THE INTEGER 2 IS FOR THE COMPLEX DECLARATION OF A.
C
      IF(IMAT.EQ.2) GO TO 100
      CALL CLEAR (A,MQ*MQ*NQ*2)
C
C     FETCH AND READ NAMELIST INPUT VARIABLES USING SUBROUTINE ASSIST
C
      CALL ASSIST
      READ(7,VALUES)
      DGTRD=3.1415/180.
      RDTDG=180./3.1415
      CLDR=CLDRG*RDTDG
      CLDA=CLDAG*RDTDG
      CNDR=CNDRG*RDTDG
      CNDA=CNDAG*RDTDG
      CYDR=CYDRG*RDTDG
      CYDA=CYDAG*RDTDG
      FTKN=6080./3600.
      VE=VPK*FTKN
      VP=VPK*FTKN
      M=GW/32.174
      ALPHA=ALPHAG*DGTRD
      LUE=0
C
C     NOTE THAT VUSER IS USED TO CALCULATE DYNAMIC PRESSURE AND MACH
C     NUMBER RATHER THAN VECTOR ELEMENTS.
C
      CALL VUSER(H,VE,LUE,MACH,QC,VP)
      UBO=COS(ALPHA)*VP
      WBO=SIN(ALPHA)*VP
      Q=QC
C
C     PRINT VARIOUS AIRPLANE CONDITIONS WHICH HELP BUILD THE A MATRIX.
C
      WRITE(6,6000) VP,M,ALPHA,Q,MACH,DGTRD,FTKN,RDTDG,UBO,WBO,CLDR,
```

Figure 7. – FORTRAN Listing of Subroutine MUSER

```
     1CLDA,CNDR,CNDA,CYDR,CYDA
C     MATRIX DEFINATION
      A(1,1,S1)=1.
      A(1,2,S1)=-IXZ/IXX
      A(1,3,S0)=-Q*S*B/IXX
      A(1,4,S0)=Q*S*B*SIN(ALPHA)/IXX
      A(2,1,S0)=-WBO
      A(2,2,S0)=UBO
      A(2,5,S1)=1.
      A(2,6,S0)=-Q*S/M
      A(2,7,S0)=-32.2
      A(3,1,S1)=-IXZ/IZZ
      A(3,2,S1)=1.
      A(3,3,S0)=-Q*S*B*SIN(ALPHA)/IZZ
      A(3,4,S0)=-Q*S*B/IZZ
      A(4,1,S0)=-1.
      A(4,7,S1)=1.
      A(4,8,S1)=-SIN(ALPHA)
      A(5,2,S0)=-1.
      A(5,8,S1)=1.
      A(6,3,S0)=1.
      A(6,9,S0)=-CLB
      A(6,10,S0)=-CLP*0.5*B/VP
      A(6,11,S0)=-CLR*0.5*B/VP
      A(6,12,S0)=-CLDR
      A(6,13,S0)=-CLDA
      A(7,6,S0)=1.
      A(7,9,S0)=-CYB
      A(7,10,S0)=-CYP*0.5*B/VP
      A(7,11,S0)=-CYR*0.5*B/VP
      A(7,12,S0)=-CYDR
      A(8,4,S0)=1.
      A(8,9,S0)=-CNB
      A( 8,10,S0)=-CNP*0.5*B/VP
      A( 8,11,S0)=-CNR*0.5*B/VP
      A( 8,12,S0)=-CNDR
      A( 8,13,S0)=-CNDA
      A( 8,09,S1)=-CNBD*0.5*B/VP
      A(09,1,S0)=-1.
      A(09,2,S0)=-SIN(ALPHA)
      A(09,10,S0)=1.
      A(10,1,S0)=SIN(ALPHA)
      A(10,2,S0)=-1.
      A(10,11,S0)=1.
      A(11,8,S2)=-57.3*0.564*K
      A(11,14,S2)=.74
      A(11,14,S1)=3.
      A(11,14,S0)=1.
      A(12,14,S0)=-1.
      A(12,15,S0)=1.
      A(12,16,S0)=1.
      A(12,17,S0)=1.
      A(13,15,S0)=-21.9
      A(13,18,S1)=1.
      A(14,17,S0)=1.
      A(14,17,S1)=.005
      A(14,18,S0)=-1.5
      A(15,16,S1)=85.
      A(15,16,S0)=1.
      A(15,17,S0)=-8.
      A(16,12,S0)=1.
      A(16,12,S1)=.05
      A(16,18,S0)=-.216
      A(17,13,S0)=1.
      A(18,5,S0)=-1./VP
      A(18,9,S0)=1.
      GO TO 900
100   A(11,14,S2)=.84
900   RETURN
6000  FORMAT(5HOVP =,F6.1,5H  H=,F9.1,9H  ALPHA=,F8.5,5H  Q=,F6.1,
     1 8H  MACH =,F5.3/
     2 8H DGTRD=,F9.6,6H FTKN=,F8.4,9H  RDTDG=,F7.3,6H UBO=,F6.1,
     3  6H WBO =,F6.1/
     4  7H CLDR =,F8.6,8H  CLDA=,F8.6,8H  CNDR =,F9.6,8H CNDA =,F11.8,
     5 8H  CYDR=,F7.5,8H  CYDA=,F11.8/1H )
      END
```

Figure 7. – (Concluded)

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SUBROUTINE VUSER

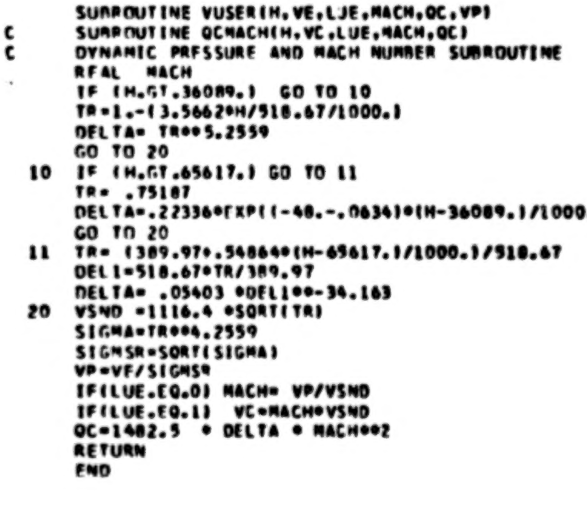
The principal purpose of subroutine VUSER is to define elements of column vector array. Thus CALL VUSER performs the same function as VECTOR or REPLACE VECTOR control cards.

```
SUBROUTINE VUSER (B, M, N, IVEC)
REAL B(M, N)
```

where:

B	is real matrix whose dimensions are M x N.
M	is the row and column dimension of the A matrix.
N	is the highest power of s plus 1.
IVEC	is a count of the number of times the CALL MUSER data card has been used.

A sample of a FORTRAN listing for subroutine VUSER is shown in figure 8.



```
      SUBROUTINE VUSER(H,VE,LJE,MACH,QC,VP)
C     SUBROUTINE QCMACH(H,VC,LUE,MACH,QC)
C     DYNAMIC PRESSURE AND MACH NUMBER SUBROUTINE
      REAL  MACH
      IF (H.GT.36089.)  GO TO 10
      TR=1.-(3.5662*H/518.67/1000.)
      DELTA= TR**5.2559
      GO TO 20
   10 IF (H.GT.65617.) GO TO 11
      TR= .75187
      DELTA=.22336*EXP((-48.-.0634)*(H-36089.)/1000./1000.)
      GO TO 20
   11 TR= (389.97+.54864*(H-65617.)/1000.)/518.67
      DEL1=518.67*TR/389.97
      DELTA= .05403 *DEL1**-34.163
   20 VSND =1116.4 *SQRT(TR)
      SIGMA=TR**4.2559
      SIGMSR=SQRT(SIGMA)
      VP=VE/SIGMSR
      IF(LUE.EQ.0) MACH= VP/VSND
      IF(LUE.EQ.1)  VC=MACH*VSND
      QC=1482.5  * DELTA * MACH**2
      RETURN
      END
```

Figure 8. – FORTRAN Listing of Subroutine VUSER

SUBROUTINE USERSB

The principal purpose of subroutine USERSB is to define elements of the matrix and/or vector. Thus CALL USERSB performs the same functions as MATRIX/REPLACE and VECTOR/REPLACEVECTOR control cards.

```
SUBROUTINE USERSB (A, B, M, N, IUSE)
COMPLEX A(M, M, N)
REAL B(M, N)
```

where:

A	is the complex matrix whose dimension are M, M, N.
B	is a real matrix whose dimensions are M x N.
M	is the row and column dimension of the A matrix.
IUSE	is the count of the number of times the **CALL USERSB** data card has been used.

A sample of a FORTRAN listing of subroutine USERSB is shown in figure 9.

```
      SUBROUTINE USERSB(A,B,MM,N,IUSE)
      COMPLEX A(MM,MM,N)
      REAL B(MM,N)
      REAL M,IYY,MW,MWD,MDE,MQ,KQ
      INTEGER S0,S1,S2,S3
      DATA S0,S1,S2,S3/1,2,3,4/
      NAMELIST/APDATA/M,IYY,Q,S,CBAR,V,CLA,CMA,CLADH,CMADH,CMQH,CLDE,
     *CLQH,CMDE,CD,GAMMA0
      IF(IUSE.GT.1)GO TO 10
      CALL CLEAR(B,MM*N)
   10 READ(5,APDATA)
      WRITE(6,APDATA)
      KQ=0.0
      TQ=0.5
      TL=0.1
      QCMD=1.0
      WGUST=0.0
      G=32.17
      RAD=57.295
      ZW= -Q*S*(RAD*CLA + CD)/(V*M)
      ZWD= -Q*S*CBAR*CLADH/(2*M*V**2)
      ZQ= -Q*S*CBAR*CLQH/(2*M*V)
      ZDE= -Q*S*(RAD*CLDE)/M
      MW= Q*S*CBAR*(RAD*CMA)/(V*IYY)
      MWD= Q*S*(CBAR**2)*CMADH/(2*IYY*V**2)
      MQ= Q*S*(CBAR**2)*CMQH/(2*V*IYY)
      MDE= Q*S*CBAR*(RAD*CMDE)/IYY
      A(1,1,S1)=(1.0-ZWD)
      A(1,1,S0)= -ZW
      A(1,2,S0)= (-V-ZQ)
      A(1,3,S0)= G*SIN(GAMMA0)
      A(1,4,S0)= -ZDE
      A(2,1,S1)= -MWD
      A(2,1,S0)= -MW
      A(2,2,S1)= 1.0
      A(2,2,S0)= -MQ
      A(2,4,S0)= -MDE
      A(3,2,S0)= -1.0
      A(3,3,S1)= 1.0
      A(4,4,S1)= 0.05
      A(4,4,S0)= 1.0
      A(4,5,S0)=-KQ
      A(5,5,S0)= 1.0
      A(5,7,S0)=-1.0
      A(6,2,S1)=-TQ
      A(6,6,S1)=TQ
      A(6,6,S0)= 1.0
      A(7,6,S0)=-1.0
      A(7,7,S1)=TL
      A(7,7,S0)= 1.0
      B(1,S0)= -ZW*WGUST
      B(2,S0)= -MW*WGUST
      B(5,S0)= -1.0*QCMD
      RETURN
```

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Figure 9. — FORTRAN Listing of Subroutine USERB

Subroutines MUSER, VUSER, and USERSB can call a number of subroutines from the QR FORTRAN library.

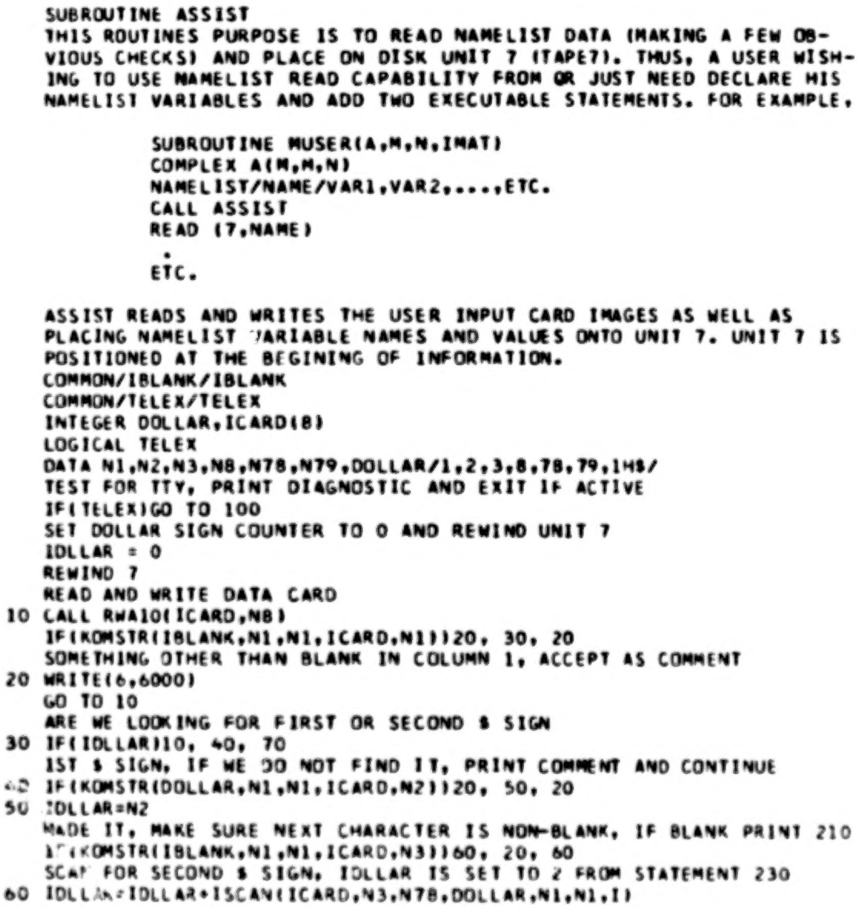
Note: The BEGIN card and its associated data must be called before the CALL MUSER, CALL VUSER, or CALL USERSB cards.

The user may read and write input and output files by using the standard FORTRAN conventions, that is, logical input is unit 5 and logical output is unit 6. The logical punch unit is 93.

Three user defined subroutines are called by MUSER, VUSER and USERSB. These subroutines perform input data and matrix manipulation functions.

SUBROUTINE ASSIST

SUBROUTINE ASSIST is called by MUSER and reads and writes card images onto disk unit 7 until a dollar sign followed by the NAMELIST name is encountered in column 2, and the NAMELIST values are terminated by a dollar sign. In addition, card images read by the QR/MUSER execution are printed as punched. The diagnostic, (THIS IS A COMMENT CARD), may appear if data appears in column one, or an infraction of the CDC 6600 NAMELIST capability occurs. A FORTRAN listing of subroutine ASSIST is shown in figure 10.



```
    SUBROUTINE ASSIST
    THIS ROUTINES PURPOSE IS TO READ NAMELIST DATA (MAKING A FEW OB-
    VIOUS CHECKS) AND PLACE ON DISK UNIT 7 (TAPE7). THUS, A USER WISH-
    ING TO USE NAMELIST READ CAPABILITY FROM QR JUST NEED DECLARE HIS
    NAMELIST VARIABLES AND ADD TWO EXECUTABLE STATEMENTS. FOR EXAMPLE,

            SUBROUTINE MUSER(A,M,N,IMAT)
            COMPLEX A(M,M,N)
            NAMELIST/NAME/VAR1,VAR2,....,ETC.
            CALL ASSIST
            READ (7,NAME)
             .
            ETC.

    ASSIST READS AND WRITES THE USER INPUT CARD IMAGES AS WELL AS
    PLACING NAMELIST VARIABLE NAMES AND VALUES ONTO UNIT 7. UNIT 7 IS
    POSITIONED AT THE BEGINING OF INFORMATION.
    COMMON/IBLANK/IBLANK
    COMMON/TELEX/TELEX
    INTEGER DOLLAR,ICARD(8)
    LOGICAL TELEX
    DATA N1,N2,N3,N8,N78,N79,DOLLAR/1,2,3,8,78,79,1H$/
    TEST FOR TTY, PRINT DIAGNOSTIC AND EXIT IF ACTIVE
    IF(TELEX)GO TO 100
    SET DOLLAR SIGN COUNTER TO 0 AND REWIND UNIT 7
    IDLLAR = 0
    REWIND 7
    READ AND WRITE DATA CARD
 10 CALL RWAIO(ICARD,N8)
    IF(KOMSTR(IBLANK,N1,N1,ICARD,N1))20, 30, 20
    SOMETHING OTHER THAN BLANK IN COLUMN 1, ACCEPT AS COMMENT
 20 WRITE(6,6000)
    GO TO 10
    ARE WE LOOKING FOR FIRST OR SECOND $ SIGN
 30 IF(IDLLAR)10, 40, 70
    1ST $ SIGN, IF WE DO NOT FIND IT, PRINT COMMENT AND CONTINUE
 40 IF(KOMSTR(DOLLAR,N1,N1,ICARD,N2))20, 50, 20
 50 IDLLAR=N2
    MADE IT, MAKE SURE NEXT CHARACTER IS NON-BLANK, IF BLANK PRINT 210
    IF(KOMSTR(IBLANK,N1,N1,ICARD,N3))60, 20, 60
    SCAN FOR SECOND $ SIGN, IDLLAR IS SET TO 2 FROM STATEMENT 230
 60 IDLLAR=IDLLAR+ISCAN(ICARD,N3,N78,DOLLAR,N1,N1,I)
```

Figure 10. – FORTRAN Listing of Subroutine ASSIST.

SUBROUTINE CLEAR, (A, N)

SUBROUTINE CLEAR zeros the first N elements of matrix A. Subroutine CLEAR is called by MUSER and USERSB. A FORTRAN listing is shown in figure 11.

SUBROUTINE NEGMAT (A, N)

SUBROUTINE NEGMAT reverses the sign of the first N elements of matrix A. Subroutine NEGMAT is called by VUSER. Subroutine NEGMAT is an entry point in subroutine CLEAR (see fig. 11).

```
C   SUBROUTINE CLEAR ZEROS THE FIRST N ELEMENTS OF MATRIX M.
      SUBROUTINE CLEAR(M,N)
      INTEGER M(1)
      COMMON/IBLANK/IBLANK
      IF(N.LE.0)GO TO 40
      DO 10 I=1,N
   10 M(I)=0
      GO TO 40
      ENTRY BLKFLL
      DO 20 I=1,N
   20 M(I)=IBLANK
      GO TO 40
C   SUBROUTINE NEGMAT REVERSES THE SIGN OF THE FIRST N ELEMENTS OF MATRIX M.
      ENTRY NEGMAT
      DO 30 I=1,N
   30 M(I)=-M(I)
   40 RETURN
      END
```

Figure 11. – FORTRAN Listing of Subroutines CLEAR and NEGMAT.

In figure 12 two cards are used to transfer to MUSER and VUSER. These subroutine calls will pass the starting position and matrix dimensions of the A and B matrices respectively. The subroutine call in VUSER to USERSB must have both calling and receiving subroutine arguments to SUBROUTINE USERSB established by the user.

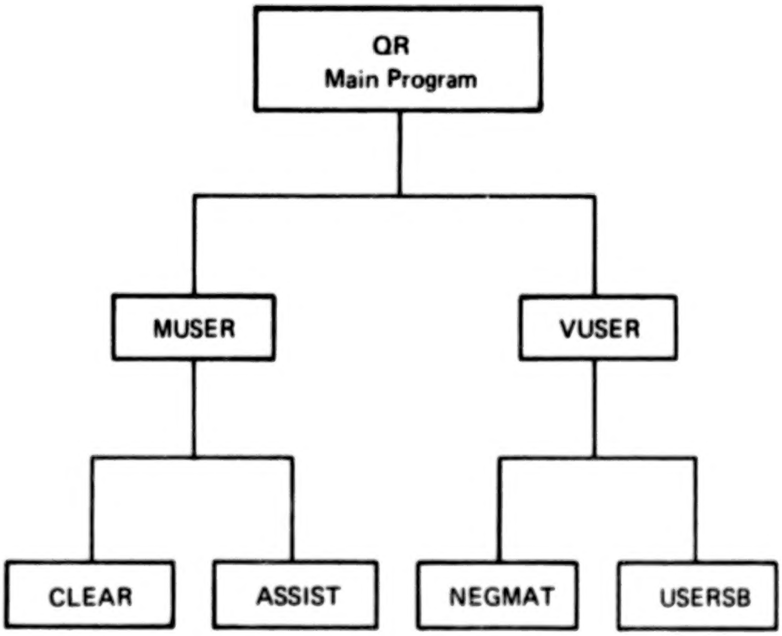


Figure 12. – Interface with Subroutines MUSER and VUSER

Figure 13 shows one supplied user subroutine, USERSB. The subroutine call to USERSB establishes the starting positions and dimensions of the A and B matrices. USERSB in turn calls subroutine CLEAR.

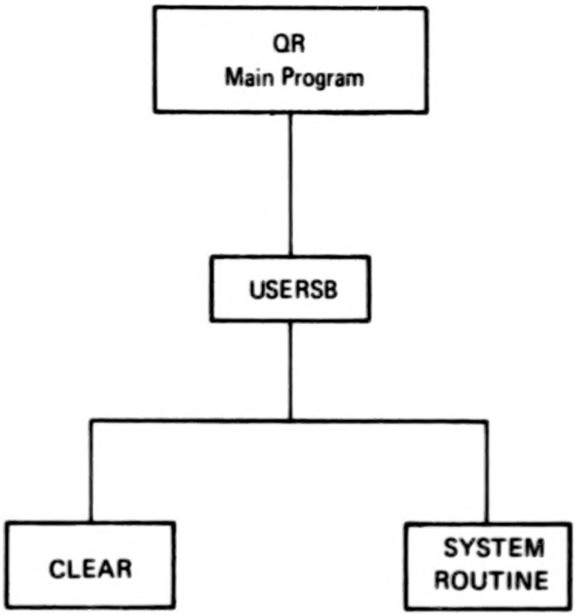

QR
Main Program
USERSB
CLEAR
SYSTEM ROUTINE

Figure 13. – Interface with Subroutine USERSB

7.0 SAMPLE PROBLEMS

7.1 EIGENVALUE CALCULATIONS

The following pages illustrate how to use the QR program to analyze the feedback control system illustrated in figure 14.

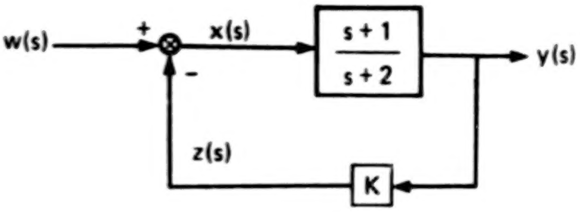


Figure 14. — A Simple Feedback Control System

Three simultaneous algabraic equations in three unknowns which result from figure 14 are:

$$x(s) = w(s) - z(s)$$

$$y(s) = \left(\frac{s+1}{s+2}\right) x(s)$$

$$z(s) = Ky(s) \tag{60}$$

Dropping the functional dependence in s the three equations may be rewritten as:

$$\begin{aligned} x \qquad\qquad + z &= w \\ (s+1)\,x - (s+2)y &= 0 \\ Ky - z &= 0 \end{aligned} \tag{61}$$

The three equations may be written in the matrix form as:

$$\begin{bmatrix} 1 & 0 & 1 \\ (s+1) & -(s+2) & 0 \\ 0 & K & -1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} w \\ 0 \\ 0 \end{Bmatrix} \tag{62}$$

The square polynomial matrix may be written as the sum of square matrices in power of s.

$$\left[\begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} s + \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & K & -1 \end{bmatrix}\right] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} w \\ 0 \\ 0 \end{Bmatrix} \tag{63}$$

To enter the square polynomial matrix as input to QR, QR requires that the matrix be in the form of equation (63). The following list of cards will result in an eigenvalue calculation with K = 1.

```
BEGIN
AN EIGENVALUE CALCULATION
     3    1
MATRIX
 2 1 1 1.0         2 2 1 -1.0        1 1 0 1.0         1 3 0 1.0
 2 1 0 1.0         2 2 0 -2.0        3 2 0 1.0         3 3 0 -1.0
(blank card)
COMPUTE
```

To perform two more eigenvalue calculation within the same execution of QR (for values of gain K = 20 and K = 0) the following list of cards follow the compute card:

```
REPLACE
 3 2 0 20.0
(blank card)
COMPUTE
REPLACE
 3 2 0 0.0
(blank card)
COMPUTE
```

After processing each COMPUTE card the current version of the square polynomial matrix resides in the computer's central memory. Therefore after the final COMPUTE is processed the value of the 3 2 0 element of the square polynomial matrix will be 0.

7.2 TIME RESPONSE CALCULATION

The following pages illustrate how to use the QR program to obtain the time response for the block diagram in figure 15 due to (1) a unit step input and (2) unit cosine input.

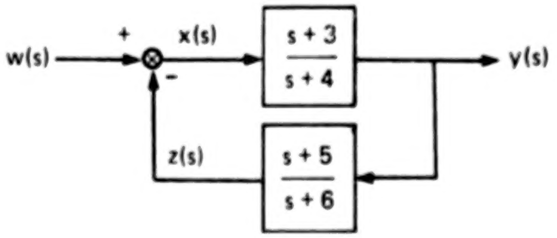


Figure 15. – A Sample Control System for Time Response Calculation

Three simultaneous algebraic equations in three unknowns which result from figure 15 are:

$$w(s) - z(s) \;=\; x(s)$$

$$x(s)\,\frac{(s+3)}{(s+4)} \;=\; y(s)$$

$$y(s)\,\frac{(s+5)}{(s+6)} \;=\; z(s) \qquad (64)$$

These three equations may be written in matrix form as:

$$\begin{bmatrix} 1 & 0 & 1 \\ s+3 & -(s+4) & 0 \\ 0 & (s+5) & -(s+6) \end{bmatrix} \left\{ \begin{matrix} x(s) \\ y(s) \\ z(s) \end{matrix} \right\} = \left\{ \begin{matrix} w(s) \\ o \\ o \end{matrix} \right\} \qquad (65)$$

Using Cramer's rule, the expression for y(s) is obtained by taking the ratio of two determinants. The numerator determinant is the determinant of the square polynomial matrix with the right-hand side of equation (65) substitute for the second column. The denominator determinant is the determinant of the square polynomial matrix.

The resulting expression is:

$$y(s) = \left(\frac{s^2 + 9s + 18}{2s^2 + 18s + 39} \right) w(s) \qquad (66)$$

where the ratio of polynomials on the right-hand side of this equation is the closed-loop transfer function. To obtain the time response of the output, first the Laplace transform of the input (for example either unit step or unit cosine) must be substituted for w(s) and then the inverse Laplace transform of the right-hand side must be obtained.

In preparing the QR - Control Card deck to obtain the time response of the output the user must:

1. Define the Laplace transform of the input w(s).

2. After substituting the Laplace transform for w(s), algebraically manipulate the first row of equation (65) to eliminate power of s from the denominator of each element in the first row.

3. Define the numerator and denominator matrices whose determinants will result in the expression in equation (66).

The data cards required for a QR time response solution to a unit step input to figure 11 are as follows:

```
BEGIN
A TIME RESPONSE CALCULATION
MATRIX
 2 1 1 1.0          2 2 1 -1.0          3 2 1 1.0          3 3 1 -1.0
 1 1 0 1.0          1 3 0 1.0           2 1 0 3.0          2 2 0 -4.0
 3 2 0 5.0          3 3 0 -6.0
(blank card)
VECTOR
 1 0 1.0
(blank card)
COMPUTE POLES
CRAMER REPLACE
     2
COMPUTE ZEROS
TIME INTERVALS
     2
0.0          .01          1.0
1.0          .1           10.
INTEGRATE
     1
FORM RATIO OF POLYNOMIALS
COMPUTE TIME RESPONSE
```

At the conclusion of the time response solution the matrix is restored into the computer memory as originally described with the **MATRIX** statement.

To calculate the time response of the system to a unit cosine forcing function (cos 5t), the Laplace transform of the forcing function is substituted for w(s) in equation (65). The denominator in the vector polynomial is cleared making the matrix polynomial a second

$$w(s) = \frac{s}{s^2 + 25}$$

order is s $(s^2 + 25$ now appears in the first row). The data cards required for a QR time solution of the system to the cosine forcing function are as follows

```
BEGIN
A TIME RESPONSE CALCULATION
     3     2
MATRIX
 2 1 1 1.0          2 2 1 -1.0          3 2 1 1.0          3 3 1 -1.0
 1 1 0 25.0         1 3 0 25.0          1 1 2 1.0          1 3 2 1.0
 2 1 0 3.0          2 2 0 -4.0          3 2 0 5.0          3 3 0 -6.0
```

```
(blank card)
VECTOR
  1  1  1.0
COMPUTE POLES
CRAMER REPLACE
      2
COMPUTE ZEROS
TIME INTERVALS
      2
0.0          .01          1.0
1.0          .1           10.0
FORM RATIO OF POLYNOMIALS
COMPUTE TIME RESPONSE
```

7.3 FREQUENCY RESPONSE AND POWER SPECTRAL DENSITY CALCULATION

The following pages illustrate how to use the QR Program to compute a frequency response function and perform a power spectral density analysis on the block diagram in figure 16.

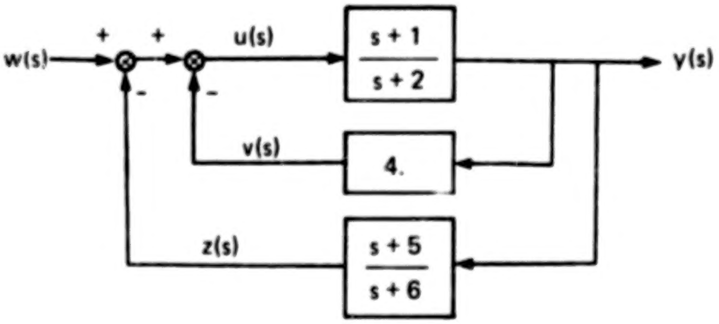


Figure 16. – A Sample Control System for Frequency Response Calculation

The four simultaneous algebraic equations in four unknowns which result from figure 16 are:

$$\begin{aligned} w(s) - z(s) - v(s) &= u(s) \\ u(s)\,\frac{(s+1)}{(s+2)} &= y(s) \\ 4y(s) &= v(s) \\ y(s)\,\frac{(s+5)}{(s+6)} &= z(s) \end{aligned} \qquad (67)$$

This four equations may be written in the matrix form as:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ (s+1) & -(s+2) & 0 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & (s+5) & 0 & -(s+6) \end{bmatrix} \begin{Bmatrix} u(s) \\ v(s) \\ y(s) \\ z(s) \end{Bmatrix} = \begin{Bmatrix} w(s) \\ o \\ o \\ o \end{Bmatrix} \qquad (68)$$

The control cards necessary to compute a frequency response function (from $\omega = 0.1$ to $\omega = 100$) for equation (68) are:

```
BEGIN
A FREQUENCY RESPONSE CALCULATION
    4     1
MATRIX
 2  1  1  1.0        2  2  1 -1.0        4  2  1  1.0        4  4  1 -1.0
 1  1  0  1.0        2  2  0 -2.0        3  2  0 4.0         3  3  0 -1.0
 4  2  0  5.0        4  4  0 -6.0        1  3  0 1.0         1  4  0 1.0
 2  1  0  1.0
(blank card)
VECTOR
 1    0 1.0
COMPUTE POLES
CRAMER REPLACE
    2
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
COMPUTE FREQUENCY RESPONSE
```

To perform a power spectral density analysis on the system represented by figure 16, the von Karman power spectrum must be defined. The following list of cards follow the COMPUTE FREQUENCY RESPONSE.

```
GUST SPECTRUM
1.0       2500.       2620.       11.0       6.0       1.0
4.78      1.79
COMPUTE PSD
```

This statement will calculate the PSD of the polynomial ratio formed prior to the frequency response calculation. In addition, this polynomial ratio is multiplied by the above described gust spectrum. At the conclusion of the frequency response solution, the matrix is restored into the computer memory as originally described with the MATRIX statement.

7.4 ROOT LOCUS CALCULATION

The following pages illustrate how to use the QR program to perform a root locus calculation for the block diagram in figure 17.

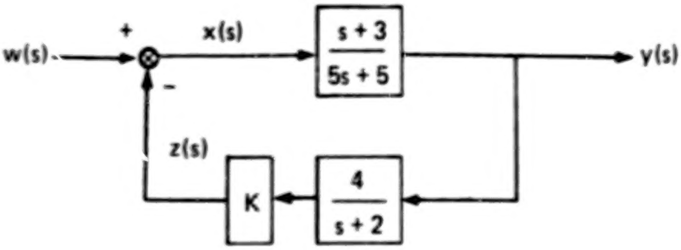


Figure 17. – A Sample Control System for Root Locus Calculations

Three simultaneous algebraic equations in three unknowns which result from figure 17 are:

$$\begin{aligned} x(s) + z(s) \quad &= w(s) \\ x(s)\,\frac{s+3}{5s+5} \quad &= y(s) \\ y(s)\,K\,\frac{4}{s+2} \quad &= z(s) \end{aligned} \tag{69}$$

This three-equation statement may be written in the matrix form as:

$$\begin{bmatrix} 1 & 0 & 1 \\ (s+3) & -(5s+5) & 0 \\ 0 & 4K & (s+2) \end{bmatrix} \begin{Bmatrix} x(s) \\ y(s) \\ z(s) \end{Bmatrix} = \begin{Bmatrix} w(s) \\ o \\ o \end{Bmatrix} \tag{70}$$

The control cards necessary to perform the Root Locus solution are:

```
BEGIN
A ROOT LOCUS CALCULATION
    3    1    2
MATRIX
 2 1 1 1.0       2 2 1 -5.0      3 3 1 -1.0      1 1 0 1.0
 1 3 0 1.0       2 1 0 3.0       2 2 0 -5.0      3 2 0 4.0
 3 3 -  -2.0
(blank card)
```

```
GAINS
     2    2
1.0       2.0
0.0       270.
     1
 3 2 0
COMPUTE ROOT LOCUS
```

The COMPUTE ROOT LOCUS statement will calculate the open-loop poles and zeros, the frequency response of their ratio, and the eigenvalues for each complex gain value. The open-loop poles and zeros that are calculated by this operation are saved in the pole and zero arrays.

Eiginvalue rootings of sample problems were hand-checked and compared with other similar program's output results. A similar procedure was followed for time response, frequency response, and flutter calculations. All results agreed to expected round-off error.

The order of the square matrix polynomial was varied over a wide range. The minimum number of rows input was 1, the maximum was 144. The degree ranged from 0 to 7. (The program will compute the determinant of a 144 by 144 square matrix polynomial of degree 0).

7.5 QR SAMPLE PROBLEM OUTPUT

The following pages contain actual printed output from the execution of the previous four examples.

```
BEGIN
7.1 EIGENVALUE CALCULATION
    3    1
MATRIX
 2 1 11.0               2 2 1-1.0               1 1 01.0                1 3 01.0
 2 1 01.0               2 2 0-2.0               3 2 01.0                3 3 0-1.0

COMPUTE
REPLACE
 3 2 020.0

COMPUTE
REPLACE
 3 2 00.0

COMPUTE
BEGIN
7.2 TIME RESPONSE CALCULATION
    3    1
MATRIX
 2 1 11.0               2 2 1-1.0               3 2 11.0                3 3 1-1.0
 1 1 11.0               1 3 11.0                2 1 03.0                2 2 0-4.0
 3 2 05.0               3 3 0-6.0

VECTOR
 1   01.0

COMPUTE POLES
CRAMER REPLACE
    2
COMPUTE ZEROS
TIME INTERVALS
    2
0.0          .01          1.0
1.0          .1           10.0
FORM RATIO OF POLYNOMIALS
COMPUTE TIME RESPONSE
BEGIN
7.2 TIME RESPONSE CALCULATION
    3    2
MATRIX
 2 1 11.0               2 2 1-1.0               3 2 11.0                3 3 1-1.0
 1 1 025.0              1 3 025.0               1 1 21.0                1 3 21.0
 2 1 03.0               2 2 0-4.0               3 2 05.0                3 3 0-6.0

VECTOR
 1   11.0

COMPUTE POLES
CRAMER REPLACE
    2
COMPUTE ZEROS
TIME INTERVALS
    2
0.0          .01          1.0
1.0          .1           10.0
FORM RATIO OF POLYNOMIALS
```

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```
COMPUTE TIME RESPONSE
BEGIN
7.3 FREQUENCY RESPONSE AND POWER SPECTRAL DENSITY CALCULATION
    4    1
MATRIX
 2 1 11.0           2 2 1-1.0           4 2 11.0            4 4 1-1.0
 1 1 01.0           2 2 0-2.0           3 2 04.0            3 3 0-1.0
 4 2 05.0           4 4 0-6.0           1 3 01.0            1 4 01.0
 2 1 01.0

VECTOR
 1   01.0

COMPUTE POLES
CRAMER REPLACE
    2
COMPUTE ZEROS
FORM RATIO OF POLYNOMIALS
FREQUENCY RESPONSE
    1    1
COMPUTE FREQUENCY RESPONSE
GUST SPECTRUM
6.4         2500.0    2620.0        11.0        6.0         1.0
4.78        1.79
PLOT PSD
    1         0    1
0.          5.0
COMPUTE PSD
BEGIN
7.4 ROOT LOCUS CALCULATION
    3    1
MATRIX
 2 1 11.0           2 2 1-5.0           3 3 1-1.0           1 1 01.0
 1 3 01.0           2 1 03.0            2 2 0-5.0           3 2 04.0
 3 3 0-2.0

GAINS
    2    2
1.0         2.0
0.0         270.0
    1
 3 2 0
COMPUTE ROOT LOCUS
STOP
```

```
BEGIN
7.1 EIGENVALUE CALCULATION
    3    1    -0    -0    -0
MATRIX
  2  1  1     1.0000000E+00       2  2  1   -1.0000000E+00  1  1  0    1.0000000E+00       1  3  0    1.0000000E+00
  2  1  0     1.0000000E+00       2  2  0   -2.0000000E+00  3  2  0    1.0000000E+00       3  3  0   -1.0000000E+00

THE INPUT MATRIX IS REAL

S** 0 MATRIX

ROW   1   1.0000E+00    .E+00       1.0000E+00
ROW   2   1.0000E+00 -2.0000E+00    .E+00
ROW   3     .E+00       1.0000E+00 -1.0000E+00

S** 1 MATRIX

ROW   1 IS ZERO.
ROW   2   1.0000E+00 -1.0000E+00    .E+00
ROW   3 IS ZERO.
COMPUTE
LEADING COEFFICIENT =  2.0000000E+00    .E+00          CP   0.199 SEC.  ROOTING SEC. =  0.002
        REAL           IMAGINARY      ZETA     OMEGA         CPS                 REAL          IMAGINARY      ZETA    OMEGA        CPS
                                                                          1      -1.50             .0          1.000   1.50       0.239
REPLACE
  3  2  0     2.0000000E+01      -0 -0 -0     -.E+00          -0 -0 -0     -.E+00           -0 -0 -0     -.E+00
COMPUTE
LEADING COEFFICIENT =  2.1000000E+01    .E+00          CP   0.209 SEC.  ROOTING SEC. =  0.003
        REAL           IMAGINARY      ZETA     OMEGA         CPS                 REAL          IMAGINARY      ZETA    OMEGA        CPS
                                                                          1      -1.05             .0          1.000   1.05       0.167
REPLACE
  3  2  0      .E+00             -0 -0 -0     -.E+00          -0 -0 -0     -.E+00           -0 -0 -0     -.E+00
COMPUTE
LEADING COEFFICIENT =  1.0000000E+G0    .E+00          CP   0.220 SEC.  ROOTING SEC. =  0.002
        REAL           IMAGINARY      ZETA     OMEGA         CPS                 REAL          IMAGINARY      ZETA    OMEGA        CPS
                                                                          1      -2.00             .0          1.000   2.00       0.318
BEGIN
7.2 TIME RESPONSE CALCULATION
    3    1    -0    -0    -0
MATRIX
  2  1  1     1.0000000E+00       2  2  1   -1.0000000E+00  3  2  1    1.0000000E+00       3  3  1   -1.0000000E+00
  1  1  1     1.0000000E+00       1  3  1    1.0000000E+00  2  1  0    3.0000000E+00       2  2  0   -4.0000000E+00
  3  2  0     5.0000000E+00       3  3  0   -6.0000000E+00 -0 -0 -0    -.E+00              -0 -0 -0    -.E+00

THE INPUT MATRIX IS REAL

S** 0 MATRIX

ROW   1 IS ZERO.
ROW   2   3.0000E+00 -4.0000E+00    .E+00
ROW   3     .E+00       5.0000E+00 -6.0000E+00

S** 1 MATRIX

ROW   1   1.0000E+00    .E+00       1.0000E+00
ROW   2   1.0000E+00 -1.0000E+00    .E+00
ROW   3     .E+00       1.0000E+00 -1.0000E+00
```

```
VECTOR
   1   0    1.0000000E+00  -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
  -0  -0    -.E+00         -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
COMPUTE POLES
LEADING COEFFICIENT =  2.0000000E+00   .E+00         CP   0.251 SEC.  ROOTING SEC. =  0.004
        REAL         IMAGINARY     ZETA    OMEGA       CPS               REAL         IMAGINARY     ZETA    OMEGA          CPS
 1      -5.37           .0         1.000   5.37     0.854          2      -3.63           .0        1.000   3.63       0.578
                                                                   3       .0             .0         .000   .0          .0
CRAMER REPLACE
    2   -0
COMPUTE ZEROS
LEADING COEFFICIENT =  1.0000000E+00   .E+00         CP   0.257 SEC.  ROOTING SEC. =  0.001
        REAL         IMAGINARY     ZETA    OMEGA       CPS               REAL         IMAGINARY     ZETA    OMEGA          CPS
 1      -6.00           .0         1.000   6.00     0.955          2      -3.00           .0        1.000   3.00       0.477
TIME INTERVALS
    2
     .E+00         1.00000E-02    1.00000E+00
    1.00000E+00    1.00000E-01    1.00000E+01
FORM RATIO OF POLYNOMIALS
********************************************************************************************************************************
CANCELLED POLES                                                  CANCELLED ZEROS

********************************************************************************************************************************
REMAINING ZEROS AFTER CANCELLATION

        REAL         IMAGINARY     ZETA    OMEGA       CPS               REAL         IMAGINARY     ZETA    OMEGA          CPS
 1      -6.00           .0         1.000   6.00     0.955          2      -3.00           .0        1.000   3.00       0.477
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  2
  5.00000E-01  4.50000E+00  9.00000E+00
********************************************************************************************************************************
REMAINING POLES AFTER CANCELLATION

        REAL         IMAGINARY     ZETA    OMEGA       CPS               REAL         IMAGINARY     ZETA    OMEGA          CPS
 1      -5.37           .0         1.000   5.37     0.854          2      -3.63           .0        1.000   3.63       0.578
                                                                   3       .0             .0         .000   .0          .0
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  3
  1.00000E+00  9.00000E+00  1.95000E+01   .E+00
********************************************************************************************************************************
COMPUTE TIME RESPONSE

  POLES                                                 RESIDUES                                *  ZEROS
  REAL    IMAGINARY MAGNITUDE  ZETA      CPS      REAL    IMAGINARY MAGNITUDE   ANGLE *  REAL     IMAGINARY MAGNITUDE  ZETA      CPS
-5.37E+00   .E+00     5.37E+00 1.000 8.54E-01   -8.07E-02    .E+00     8.07E-02   180.0 * -6.00E+00    .E+00      6.00E+00 1.000 9.55E-01
-3.63E+00   .E+00     3.63E+00 1.000 5.78E-01    1.19E-01    .E+00     1.19E-01      .0 * -3.00E+00    .E+00      3.00E+00 1.000 4.77E-01
 -.E+00    -.E+00      .E+00    .000  .E+00      4.62E-01    .E+00     4.62E-01      .0 *
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     TIME        RESPONSE        TIME        RESPONSE        TIME        RESPONSE        TIME        RESPONSE        TIME        RESPONSE

   .0            0.5000       1.0000E-02     0.5000       2.0000E-02     0.4999       3.0000E-02     0.4997       4.0000E-02     0.4995
 5.0000E-02      0.4992       6.0000E-02     0.4989       7.0000E-02     0.4985       8.0000E-02     0.4981       9.0000E-02     0.4977
 0.1000          0.4972       0.1100         0.4967       0.1200         0.4962       0.1300         0.4957       0.1400         0.4951
 0.1500          0.4945       0.1600         0.4940       0.1700         0.4934       0.1800         0.4928       0.1900         0.4922
 0.2000          0.4916       0.2100         0.4909       0.2200         0.4903       0.2300         0.4897       0.2400         0.4891
 0.2500          0.4885       0.2600         0.4879       0.2700         0.4873       0.2800         0.4867       0.2900         0.4861
 0.3000          0.4855       0.3100         0.4849       0.3200         0.4843       0.3300         0.4837       0.3400         0.4832
 0.3500          0.4826       0.3600         0.4821       0.3700         0.4815       0.3800         0.4810       0.3900         0.4805
 0.4000          0.4800       0.4100         0.4795       0.4200         0.4790       0.4300         0.4785       0.4400         0.4780
 0.4500          0.4775       0.4600         0.4771       0.4700         0.4767       0.4800         0.4762       0.4900         0.4758
 0.5000          0.4754       0.5100         0.4750       0.5200         0.4746       0.5300         0.4742       0.5400         0.4738
 0.5500          0.4735       0.5600         0.4731       0.5700         0.4728       0.5800         0.4724       0.5900         0.4721
 0.6000          0.4718       0.6100         0.4715       0.6200         0.4712       0.6300         0.4709       0.6400         0.4706
 0.6500          0.4703       0.6600         0.4700       0.6700         0.4698       0.6800         0.4695       0.6900         0.4693
 0.7000          0.4690       0.7100         0.4688       0.7200         0.4686       0.7300         0.4683       0.7400         0.4681
 0.7500          0.4679       0.7600         0.4677       0.7700         0.4675       0.7800         0.4673       0.7900         0.4671
 0.8000          0.4669       0.8100         0.4668       0.8200         0.4666       0.8300         0.4664       0.8400         0.4663
 0.8500          0.4661       0.8600         0.4660       0.8700         0.4658       0.8800         0.4657       0.8900         0.4656
 0.9000          0.4654       0.9100         0.4653       0.9200         0.4652       0.9300         0.4650       0.9400         0.4649
 0.9500          0.4648       0.9600         0.4647       0.9700         0.4646       0.9800         0.4645       0.9900         0.4644
  1.000          0.4643        1.100         0.4635        1.200         0.4629        1.300         0.4625        1.400         0.4622
  1.500          0.4620        1.600         0.4619        1.700         0.4618        1.800         0.4617        1.900         0.4617
  2.000          0.4616        2.100         0.4616        2.200         0.4616        2.300         0.4616        2.400         0.4616
  2.500          0.4616        2.600         0.4615        2.700         0.4615        2.800         0.4615        2.900         0.4615
  3.000          0.4615        3.100         0.4615        3.200         0.4615        3.300         0.4615        3.400         0.4615
  3.500          0.4615        3.600         0.4615        3.700         0.4615        3.800         0.4615        3.900         0.4615
  4.000          0.4615        4.100         0.4615        4.200         0.4615        4.300         0.4615        4.400         0.4615
  4.500          0.4615        4.600         0.4615        4.700         0.4615        4.800         0.4615        4.900         0.4615
  5.000          0.4615        5.100         0.4615        5.200         0.4615        5.300         0.4615        5.400         0.4615
  5.500          0.4615        5.600         0.4615        5.700         0.4615        5.800         0.4615        5.900         0.4615
  6.000          0.4615        6.100         0.4615        6.200         0.4615        6.300         0.4615        6.400         0.4615
  6.500          0.4615        6.600         0.4615        6.700         0.4615        6.800         0.4615        6.900         0.4615
  7.000          0.4615        7.100         0.4615        7.200         0.4615        7.300         0.4615        7.400         0.4615
  7.500          0.4615        7.600         0.4615        7.700         0.4615        7.800         0.4615        7.900         0.4615
  8.000          0.4615        8.100         0.4615        8.200         0.4615        8.300         0.4615        8.400         0.4615
  8.500          0.4615        8.600         0.4615        8.700         0.4615        8.800         0.4615        8.900         0.4615
  9.000          0.4615        9.100         0.4615        9.200         0.4615        9.300         0.4615        9.400         0.4615
  9.500          0.4615        9.600         0.4615        9.700         0.4615        9.800         0.4615        9.900         0.4615
  10.00          0.4615
BEGIN
7.2 TIME RESPONSE CALCULATION
    3    2   -0   -0   -0
MATRIX
  2  1  1     1.0000000E+00        2  2  1   -1.0000000E+00  3  2  1     1.0000000E+00       3  3  1   -1.0000000E+00
  1  1  0     2.5000000E+01        1  3  0    2.5000000E+01  1  1  2     1.0000000E+00       1  3  2    1.0000000E+00
  2  1  0     3.0000000E+00        2  2  0   -4.0000000E+00  3  2  0     5.0000000E+00       3  3  0   -6.0000000E+00

THE INPUT MATRIX IS REAL

S** 0 MATRIX

ROW   1   2.5000E+01    .E+00       2.5000E+01
ROW   2   3.0000E+00 -4.0000E+00    .E+00
ROW   3    .E+00       5.0000E+00 -6.0000E+00

S** 1 MATRIX
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      TIME        RESPONSE        TIME       RESPONSE        TIME        RESPONSE        TIME        RESPONSE        TIME        RESPONSE

     .0           0.5000      1.0000E-02    0.4993      2.0000E-02     0.4974     3.0000E-02     0.4941      4.0000E-02     0.4895
  5.0000E-02      0.4837      6.0000E-02    0.4765      7.0000E-02     0.4682     8.0000E-02     0.4587      9.0000E-02     0.4479
  0.1000          0.4361      0.1100        0.4231      0.1200         0.4090     0.1300         0.3939      0.1400         0.3778
  0.1500          0.3607      0.1600        0.3427      0.1700         0.3238     0.1800         0.3042      0.1900         0.2837
  0.2000          0.2626      0.2100        0.2407      0.2200         0.2183     0.2300         0.1954      0.2400         0.1719
  0.2500          0.1480      0.2600        0.1238      0.2700         9.9216E-02 0.2800         7.4417E-02  0.2900         4.9437E-02
  0.3000          2.4340E-02  0.3100       -8.1219E-04  0.3200        -2.5955E-02 0.3300        -5.1026E-02  0.3400        -7.5961E-02
  0.3500         -0.1007      0.3600       -0.1252      0.3700        -0.1493     0.3800        -0.1731      0.3900        -0.1964
  0.4000         -0.2193      0.4100       -0.2415      0.4200        -0.2632     0.4300        -0.2842      0.4400        -0.3045
  0.4500         -0.3240      0.4600       -0.3426      0.4700        -0.3605     0.4800        -0.3774      0.4900        -0.3933
  0.5000         -0.4083      0.5100       -0.4222      0.5200        -0.4351     0.5300        -0.4468      0.5400        -0.4575
  0.5500         -0.4670      0.5600       -0.4753      0.5700        -0.4824     0.5800        -0.4883      0.5900        -0.4929
  0.6000         -0.4964      0.6100       -0.4986      0.6200        -0.4995     0.6300        -0.4991      0.6400        -0.4976
  0.6500         -0.4947      0.6600       -0.4906      0.6700        -0.4853     0.6800        -0.4788      0.6900        -0.4710
  0.7000         -0.4621      0.7100       -0.4520      0.7200        -0.4408     0.7300        -0.4285      0.7400        -0.4151
  0.7500         -0.4006      0.7600       -0.3852      0.7700        -0.3687     0.7800        -0.3514      0.7900        -0.3331
  0.8000         -0.3140      0.8100       -0.2942      0.8200        -0.2736     0.8300        -0.2523      0.8400        -0.2303
  0.8500         -0.2078      0.8600       -0.1848      0.8700        -0.1613     0.8800        -0.1374      0.8900        -0.1131
  0.9000         -8.8549E-02  0.9100       -6.3776E-02  0.9200        -3.8839E-02 0.9300        -1.3801E-02  0.9400         1.1275E-02
  0.9500          3.6327E-02  0.9600        6.1291E-02  0.9700         8.6106E-02 0.9800         0.1107      0.9900         0.1350
   1.000          0.1590       1.100        0.3680       1.200         0.4871      1.300         0.4870       1.400         0.3679
   1.500          0.1588       1.600       -8.9174E-02   1.700        -0.3152      1.800        -0.4641       1.900        -0.4993
   2.000         -0.4123       2.100       -0.2243       2.200         1.8656E-02  2.300         0.2570       2.400         0.4324
   2.500          0.5020       2.600        0.4486       2.700         0.2855      2.800         5.2377E-02   2.900        -0.1935
   3.000         -0.3920       3.100       -0.4946       3.200        -0.4760      3.300        -0.3409       3.400        -0.1223
   3.500          0.1262       3.600        0.3438       3.700         0.4773      3.800         0.4939       3.900         0.3896
   4.000          0.1899       4.100       -5.6311E-02   4.200        -0.2887      4.300        -0.4504       4.400        -0.5018
   4.500         -0.4304       4.600       -0.2536       4.700        -1.4687E-02  4.800         0.2278       4.900         0.4145
   5.000          0.4998       5.100        0.4626       5.200         0.3122      5.300         8.5391E-02   5.400        -0.1624
   5.500         -0.3704       5.600       -0.4877       5.700        -0.4856      5.800        -0.3646       5.900        -0.1544
   6.000          9.3660E-02   6.100        0.3188       6.200         0.4658      6.300         0.4989       6.400         0.4097
   6.500          0.2203       6.600       -2.3085E-02   6.700        -0.2608      6.800        -0.4347       6.900        -0.5021
   7.000         -0.4466       7.100       -0.2818       7.200        -4.7952E-02  7.300         0.1976       7.400         0.3948
   7.500          0.4953       7.600        0.4746       7.700         0.3376      7.800         0.1180       7.900        -0.1305
   8.000         -0.3470       8.100       -0.4786       8.200        -0.4930      8.300        -0.3867       8.400        -0.1857
   8.500          6.0726E-02   8.600        0.2923       8.700         0.4524      8.800         0.5016       8.900         0.4281
   9.000          0.2497       9.100        1.0243E-02   9.200        -0.2318      9.300        -0.4170       9.400        -0.5002
   9.500         -0.4609       9.600       -0.3087       9.700        -8.1006E-02  9.800         0.1666       9.900         0.3733
   10.00          0.4887
BEGIN
7.3 FREQUENCY RESPONSE AND POWER SPECTRAL DENSITY CALCULATION
    4    1   -0   -0   -0
MATRIX
   2  1  1     1.0000000E+00       2  2  1    -1.0000000E+00  4  2  1     1.0000000E+00       4  4  1   -1.0000000E+00
   1  1  0     1.0000000E+00       2  2  0    -2.0000000E+00  3  2  0     4.0000000E+00       3  3  0   -1.0000000E+00
   4  2  0     5.0000000E+00       4  4  0    -6.0000000E+00  1  3  0     1.0000000E+00       1  4  0    1.0000000E+00
   2  1  0     1.0000000E+00      -0 -0 -0     -.E+00        -0 -0 -0     -.E+00             -0 -0 -0    -.E+00

THE INPUT MATRIX IS REAL

S** O MATRIX

ROW   1   1.0000E+00    .E+00        1.0000E+00  1.0000E+00
ROW   2   1.0000E+00 -2.0000E+00    .E+00         .E+00
ROW   3    .E+00      4.0000E+00 -1.0000E+00    .E+00
ROW   4    .E+00      5.0000E+00    .E+00      -6.0000E+00
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S** 1 MATRIX

ROW   1 IS ZERO.
ROW   2   1.0000E+00 -1.0000E+00    .E+00       .E+00
ROW   3 IS ZERO.
ROW   4    .E+00      1.0000E+00    .E+00     -1.0000E+00
VECTOR
   1   0    1.0000000E+00  -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
   -0  -0   -.E+00         -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
COMPUTE POLES
LEADING COEFFICIENT = -6.0000000E+00    .E+00         CP    0.589 SEC.  ROOTING SEC. =  0.005
        REAL         IMAGINARY      ZETA     OMEGA       CPS                REAL         IMAGINARY      ZETA     OMEGA       CPS
 1      -5.83           .0          1.000   5.83      0.927          2     -1.17            .0          1.000   1.17      0.187
CRAMER REPLACE
    2   -0
COMPUTE ZEROS
LEADING COEFFICIENT = -1.0000000E+00    .E+00         CP    0.597 SEC.  ROOTING SEC. =  0.002
        REAL         IMAGINARY      ZETA     OMEGA       CPS                REAL         IMAGINARY      ZETA     OMEGA       CPS
 1      -6.00           .0          1.000   6.00      0.955          2     -1.00            .0          1.000   1.00      0.159
FORM RATIO OF POLYNOMIALS
************************************************************************************************************************************
CANCELLED POLES                                                    CANCELLED ZEROS

************************************************************************************************************************************
REMAINING ZEROS AFTER CANCELLATION

        REAL         IMAGINARY      ZETA     OMEGA       CPS                REAL         IMAGINARY      ZETA     OMEGA       CPS
 1      -6.00           .0          1.000   6.00      0.955          2     -1.00            .0          1.000   1.00      0.159
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  2
  1.66667E-01  1.16667E+00  1.00000E+00
************************************************************************************************************************************
REMAINING POLES AFTER CANCELLATION

        REAL         IMAGINARY      ZETA     OMEGA       CPS                REAL         IMAGINARY      ZETA     OMEGA       CPS
 1      -5.83           .0          1.000   5.83      0.927          2     -1.17            .0          1.000   1.17      0.187
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  2
  1.00000E+00  7.00000E+00  6.83333E+00
************************************************************************************************************************************
FREQUENCY RESPONSE
    1    1
COMPUTE FREQUENCY RESPONSE
MEAN SQUARE VALUE =  7.880E+02, RMS =  2.807E+01
CHARACTERISTIC FREQUENCY =  8.296E+03 CPS (5.212E+04 RADIAN/SECOND)
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141

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ROW    1 IS ZERO.
ROW    2   1.0000E+00 -1.0000E+00    .E+00
ROW    3    .E+00      1.0000E+00 -1.0000E+00

S** 2 MATRIX

ROW    1   1.0000E+00   .E+00      1.0000E+00
ROW    2 IS ZERO.
ROW    3 IS ZERO.
VECTOR
   1   1    1.0000000E+00  -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
  -0  -0    -.E+00         -0  -0    -.E+00          -0  -0    -.E+00          -0  -0    -.E+00
COMPUTE POLES
LEADING COEFFICIENT =  2.0000000E+00    .E+00         CP   0.411 SEC.  ROOTING SEC. =   0.004
        REAL         IMAGINARY      ZETA     OMEGA       CPS               REAL          IMAGINARY     ZETA     OMEGA       CPS
 1      -5.37           .0          1.000   5.37      0.854          3      .0              5.00       -.000   5.00      0.796
 2       .0            -5.00        -.000   5.00      0.796          4     -3.63             .0        1.000   3.63      0.578
CRAMER REPLACE
    2   -0
COMPUTE ZEROS
LEADING COEFFICIEN  =  1.0000000E+00    .E+00         CP   0.419 SEC.  ROOTING SEC. =   0.002
        REAL         IMAGINARY      ZETA     OMEGA       CPS               REAL          IMAGINARY     ZETA     OMEGA       CPS
 1      -6.00           .0          1.000   6.00      0.955          2     -3.00             .0        1.000   3.00      0.477
                                                                     3      .0               .0         .000   .0         .0
TIME INTERVALS
    2
     .E+00         1.00000E-02    1.00000E+00
    1.00000E+00    1.00000E-01    1.00000E+01
FORM RATIO OF POLYNOMIALS
************************************************************************************************************************************
CANCELLED POLES                                                    CANCELLED ZEROS

************************************************************************************************************************************
REMAINING ZEROS AFTER CANCELLATION

        REAL         IMAGINARY      ZETA     OMEGA       CPS               REAL          IMAGINARY     ZETA     OMEGA       CPS
 1      -6.00           .0          1.000   6.00      0.955          2     -3.00             .0        1.000   3.00      0.477
                                                                     3      .0               .0         .000   .0         .0
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  3
  5.00000E-01  4.50000E+00  9.00000E+00   .E+00
************************************************************************************************************************************
REMAINING POLES AFTER CANCELLATION

        REAL         IMAGINARY      ZETA     OMEGA       CPS               REAL          IMAGINARY     ZETA     OMEGA       CPS
 1      -5.37           .0          1.000   5.37      0.854          3      .0              5.00       -.000   5.00      0.796
 2       .0            -5.00        -.000   5.00      0.796          4     -3.63             .0        1.000   3.63      0.578
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  4
  1.00000E+00  9.00000E+00  4.45000E+01  2.25000E+02  4.87500E+02
************************************************************************************************************************************
COMPUTE TIME RESPONSE

  POLES                                           RESIDUES                         *    ZEROS
  REAL    IMAGINARY MAGNITUDE  ZETA      CPS      REAL IMAGINARY  MAGNITUDE  ANGLE *    REAL  IMAGINARY   MAGNITUDE  ZETA      CPS
-5.37E+00     .E+00  5.37E+00 1.000 8.54E-01  -4.32E-02    .E+00   4.32E-02  180.0 * -6.00E+00    .E+00    6.00E+00 1.000 9.55E-01
-3.63E+00     .E+00  3.63E+00 1.000 5.78E-01   4.12E-02    .E+00   4.12E-02     .0 * -3.00E+00    .E+00    3.00E+00 1.000 4.77E-01
    .E+00 -5.00E+00  5.00E+00 -.000 7.96E-01   2.51E-01 -8.21E-03  2.51E-01   -1.9 *     .E+00    .E+00       .E+00  .000   .E+00
    .E+00  5.00E+00  5.00E+00 -.000 7.96E-01   2.51E-01  8.21E-03  2.51E-01    1.9 *
   FREQUENCY        AMPLITUDE    DB  PHASE       FREQUENCY        AMPLITUDE    DB  PHASE       FREQUENCY        AMPLITUDE    DB  PHASE
(RAD/SEC)  (CPS)      RATIO          (DEG)   (RAD/SEC)  (CPS)      RATIO          (DEG)   (RAD/SEC)  (CPS)      RATIO          (DEG)

 .E+00      .E+00    1.46E-01 -16.7     .0 *  3.10E-01  4.93E-02  1.48E-01 -16.6    2.3 *  2.04E+02  3.24E+01  1.67E-01 -15.6     0.0
1.00E-03  1.59E-04  1.46E-01 -16.7    0.0 *  4.65E-01  7.40E-02  1.50E-01 -16.5    3.2 *  3.05E+02  4.86E+01  1.67E-01 -15.6     0.0
1.06E-03  1.69E-04  1.46E-01 -16.7    0.0 *  6.97E-01  1.11E-01  1.53E-01 -16.3    4.0 *  4.58E+02  7.29E+01  1.67E-01 -15.6     0.0
1.59E-03  2.53E-04  1.46E-01 -16.7    0.0 *  1.05E+00  1.66E-01  1.58E-01 -16.0    4.3 *  6.87E+02  1.09E+02  1.67E-01 -15.6     0.0
2.39E-03  3.80E-04  1.46E-01 -16.7    0.0 *  1.57E+00  2.50E-01  1.63E-01 -15.8    3.8 *  1.03E+03  1.64E+02  1.67E-01 -15.6     0.0
3.58E-03  5.70E-04  1.46E-01 -16.7    0.0 *  2.35E+00  3.74E-01  1.66E-01 -15.6    2.9 *  1.55E+03  2.46E+02  1.67E-01 -15.6     0.0
5.37E-03  8.55E-04  1.46E-01 -16.7    0.0 *  3.53E+00  5.62E-01  1.68E-01 -15.5    1.8 *  2.32E+03  3.69E+02  1.67E-01 -15.6     0.0
8.06E-03  1.28E-03  1.46E-01 -16.7    0.1 *  5.29E+00  8.43E-01  1.68E-01 -15.5    1.0 *  3.48E+03  5.53E+02  1.67E-01 -15.6     0.0
1.21E-02  1.92E-03  1.46E-01 -16.7    0.1 *  7.94E+00  1.26E+00  1.68E-01 -15.5    0.4 *  5.22E+03  8.30E+02  1.67E-01 -15.6     0.0
1.81E-02  2.89E-03  1.46E-01 -16.7    0.1 *  1.19E+01  1.90E+00  1.67E-01 -15.5    0.2 *  7.82E+03  1.25E+03  1.67E-01 -15.6     0.0
2.72E-02  4.33E-03  1.46E-01 -16.7    0.2 *  1.79E+01  2.84E+00  1.67E-01 -15.5    0.1 *  1.17E+04  1.87E+03  1.67E-01 -15.6     0.0
4.08E-02  6.49E-03  1.46E-01 -16.7    0.3 *  2.68E+01  4.27E+00  1.67E-01 -15.6    0.0 *  1.76E+04  2.80E+03  1.67E-01 -15.6     0.0
6.12E-02  9.74E-03  1.46E-01 -16.7    0.5 *  4.02E+01  6.40E+00  1.67E-01 -15.6    0.0 *  2.64E+04  4.20E+03  1.67E-01 -15.6     0.0
9.18E-02  1.46E-02  1.47E-01 -16.7    0.7 *  6.03E+01  9.60E+00  1.67E-01 -15.6    0.0 *  3.96E+04  6.30E+03  1.67E-01 -15.6     0.0
1.38E-01  2.19E-02  1.47E-01 -16.7    1.1 *  9.04E+01  1.44E+01  1.67E-01 -15.6    0.0 *  5.94E+04  9.46E+03  1.67E-01 -15.6     0.0
2.07E-01  3.29E-02  1.47E-01 -16.6    1.6 *  1.36E+02  2.16E+01  1.67E-01 -15.6    0.0 *  8.91E+04  1.42E+04  1.67E-01 -15.6     0.0

            * * *  M I N   V A L U E S  * * *                            * * *  M A X   V A L U E S  * * *

    OMEGA        CPS       AMPLITUDE      DB       PHASE              OMEGA       CPS       AMPLITUDE      DB       PHASE

                                                                    5.66E+00   9.01E-01   1.68E-01    -15.5      0.8 CW
GUST SPECTRUM
    6.40000E+00     2.50000E+03     6.20000E+02     1.10000E+01     6.00000E+00     1.00000E+00
    4.78000E+00     1.79000E+00
PLOT PSD
    1    -0     0     1
     .E+00          5.00000E+00
COMPUTE PSD
MEAN SQUARE VALUE =  9.916E-01, RMS =  9.958E-01
CHARACTERISTIC FREQUENCY =  1.057E+02 CPS (6.642E+02 RADIAN/SECOND)
```

```
    FREQUENCY            PSD        DB PERCENT      FREQUENCY            PSD        DB PERCENT      FREQUENCY            PSD        DB PERCENT
(RAD/SEC)   (CPS)                         OF MS  (RAD/SEC)   (CPS)                        OF MS  (RAD/SEC)   (CPS)                        OF MS

  .E+00      .E+00    7.07E+00    8.5      .0 *  1.62E-01  2.58E-02  7.65E+00    8.8   19.7 *  4.83E+01  7.68E+00  2.31E-03 -26.4   97.2
1.85E-04  2.95E-05    7.07E+00    8.5     0.0 *  2.43E-01  3.87E-02  6.42E+00    8.1   28.9 *  6.36E+01  1.01E+01  1.46E-03 -28.4   97.7
2.47E-04  3.93E-05    7.07E+00    8.5     0.0 *  3.65E-01  5.80E-02  4.55E+00    6.6   39.5 *  9.54E+01  1.52E+01  7.41E-04 -31.3   98.2
3.70E-04  5.89E-05    7.07E+00    8.5     0.0 *  5.47E-01  8.70E-02  2.84E+00    4.5   50.3 *  1.43E+02  2.28E+01  3.77E-04 -34.2   98.6
5.55E-04  8.83E-05    7.07E+00    8.5     0.1 *  8.14E-01  1.30E-01  1.67E+00    2.2   60.0 *  2.15E+02  3.42E+01  1.92E-04 -37.2   99.0
8.33E-04  1.33E-04    7.07E+00    8.5     0.1 *  1.16E+00  1.85E-01  1.01E+00    0.1   67.4 *  3.22E+02  5.13E+01  9.76E-05 -40.1   99.2
1.25E-03  1.99E-04    7.07E+00    8.5     0.1 *  1.61E+00  2.57E-01  6.24E-01   -2.0   73.4 *  4.83E+02  7.69E+01  4.97E-05 -43.0   99.4
1.87E-03  2.98E-04    7.07E+00    8.5     0.2 *  2.21E+00  3.52E-01  3.85E-01   -4.1   78.3 *  7.25E+02  1.15E+02  2.53E-05 -46.0   99.6
2.81E-03  4.47E-04    7.08E+00    8.5     0.3 *  3.00E+00  4.77E-01  2.38E-01   -6.2   82.2 *  1.09E+03  1.73E+02  1.29E-05 -48.9   99.7
4.22E-03  6.71E-04    7.08E+00    8.5     0.5 *  4.01E+00  6.39E-01  1.48E-01   -8.3   85.3 *  1.63E+03  2.59E+02  6.54E-06 -51.8   99.8
6.32E-03  1.01E-03    7.08E+00    8.5     0.7 *  5.33E+00  8.48E-01  9.24E-02  -10.3   87.9 *  2.45E+03  3.89E+02  3.33E-06 -54.8   99.8
9.48E-03  1.51E-03    7.09E+00    8.5     1.1 *  7.04E+00  1.12E+00  5.80E-02  -12.4   89.9 *  3.67E+03  5.84E+02  1.69E-06 -57.7   99.9
1.42E-02  2.26E-03    7.11E+00    8.5     1.6 *  9.28E+00  1.48E+00  3.65E-02  -14.4   91.6 *  5.50E+03  8.76E+02  8.61E-07 -60.6   99.9
2.13E-02  3.40E-03    7.15E+00    8.5     2.4 *  1.22E+01  1.94E+00  2.30E-02  -16.4   93.0 *  8.25E+03  1.31E+03  4.38E-07 -63.6   99.9
3.20E-02  5.09E-03    7.24E+00    8.6     3.7 *  1.61E+01  2.56E+00  1.45E-02  -18.4   94.2 *  1.24E+04  1.97E+03  2.23E-07 -66.5  100.0
4.80E-02  7.64E-03    7.41E+00    8.7     5.5 *  2.11E+01  3.36E+00  9.18E-03  -20.4   95.2 *  1.86E+04  2.96E+03  1.13E-07 -69.5  100.0
7.20E-02  1.15E-02    7.68E+00    8.9     8.5 *  2.78E+01  4.43E+00  5.79E-03  -22.4   96.0 *  2.79E+04  4.43E+03  5.77E-08 -72.4  100.0
1.08E-01  1.72E-02    7.92E+00    9.0    13.0 *  3.66E+01  5.83E+00  3.66E-03  -24.4   96.6 *  4.18E+04  6.65E+03  2.94E-08 -75.3  100.0
                                                                                                6.27E+04  9.97E+03  1.49E-08 -78.3  100.0

THE NUMBER OF POINTS OUTSIDE THE PLOT =  27

          * * *  M I N   V A L U E S  * * *                               * * *  M A X   V A L U E S  * * *

    OMEGA       CPS      AMPLITUDE      DB      PHASE              OMEGA      CPS       AMPLITUDE      DB      PHASE

                                                                1.16E-01   1.84E-02   7.92E+00      9.0     0.9 CCW
BEGIN
7.4 ROOT LOCUS CALCULATION
    3    1    -0    -0    -0
MATRIX
  2  1  1     1.0000000E+00      2  2  1   -5.0000000E+00  3  3  1   -1.0000000E+00      1  1  0    1.0000000E+00
  1  3  0     1.0000000E+00      2  1  0    3.0000000E+00  2  2  0   -5.0000000E+00      3  2  0    4.0000000E+00
  3  3  0    -2.0000000E+00     -0 -0 -0    -.E+00        -0 -0 -0    -.E+00            -0 -0 -0    -.E+00

THE INPUT MATRIX IS REAL

S** 0 MATRIX

ROW   1   1.0000E+00    .E+00        1.0000E+00
ROW   2   3.0000E+00 -5.0000E+00     .E+00
ROW   3    .E+00       4.0000E+00 -2.0000E+00

S** 1 MATRIX

ROW   1 IS ZERO.
ROW   2   1.0000E+00 -5.0000E+00     .E+00
ROW   3    .E+00        .E+00       -1.0000E+00
GAINS
    2    2
    1.00000E+00      2.00000E+00
     .E+00           2.70000E+02
    1
 3 2 0
COMPUTE ROOT LOCUS
```

7.4 ROOT LOCUS CALCULATION

```
                                                  MATRIX ORDER =  3

                                             DEGREE OF MATRIX POLYNOMIAL =  1
ZEROS (NUMERATOR ROOTS)
LEADING COEFFICIENT =  4.0000000E+00    .E+00         CP   0.772 SEC.  ROOTING SEC. =  0.002
        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
                                                                   1      -3.00           .0          1.000   3.00      0.477
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  1
  4.00000E+00  1.20000E+01
POLES (DENOMINATOR ROOTS)
LEADING COEFFICIENT =  5.0000000E+00    .E+00         CP   0.779 SEC.  ROOTING SEC. =  0.001
        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
 1      -2.00           .0          1.000   2.00      0.318         2      -1.00           .0          1.000   1.00      0.159
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  2
  5.00000E+00  1.50000E+01  1.00000E+01
**********************************************************************************************************************************
CANCELLED POLES                                                   CANCELLED ZEROS

**********************************************************************************************************************************
REMAINING ZEROS AFTER CANCELLATION

        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
                                                                   1      -3.00           .0          1.000   3.00      0.477
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  1
  8.00000E-01  2.40000E+00
**********************************************************************************************************************************
REMAINING POLES AFTER CANCELLATION

        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
 1      -2.00           .0          1.000   2.00      0.318         2      -1.00           .0          1.000   1.00      0.159
POLYNOMIAL COEFFICIENTS, FIRST COEFFICIENTS ORDER =  2
  1.00000E+00  3.00000E+00  2.00000E+00
**********************************************************************************************************************************
MEAN SQUARE VALUE =  5.906E-01, RMS =  7.685E-01
CHARACTERISTIC FREQUENCY =  1.683E+01 CPS (1.058E+02 RADIAN/SECOND)

               *** P H A S E   M A R G I N ***

                                      PHASE      DIRECTION OF TRAVEL
        OMEGA       CPS       PHASE  MARGIN      AMPLITUDE     PHASE
       6.09E-01   9.70E-02    323.2   143.2      DECREASE      CW
GAIN =     1.00        PHASE =    .0 (DEGREES) LEADING COEFFICIENT =  1.000E+00    .E+00     CP   0.824 SEC.  ROOTING SEC. =   .000
        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
 1      -1.90         -0.889        0.906   2.10      0.334         2      -1.90          0.889         0.906   2.10      0.334


GAIN =     2.00        PHASE =    .0 (DEGREES) LEADING COEFFICIENT =  1.000E+00    .E+00     CP   0.829 SEC.  ROOTING SEC. =  0.001
        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
 1      -2.30          -1.23        0.882   2.61      0.415         2      -2.30           1.23         0.882   2.61      0.415


GAIN =     1.00        PHASE =270.0 (DEGREES) LEADING COEFFICIENT =  1.000E+00    .E+00     CP   0.835 SEC.  ROOTING SEC. =   .000
        REAL         IMAGINARY     ZETA      OMEGA      CPS               REAL         IMAGINARY     ZETA     OMEGA       CPS
 1      -2.30         -0.346        0.989   2.33      0.371         2      -0.696          1.15         0.519   1.34      0.213
```

```
GAIN =      2.00       PHASE =270.0 (DEGREES) LEADING COEFFICIENT =  1.000E+00   .E+00     CP   0.840 SEC.  ROOTING SEC. =  0.001
        REAL           IMAGINARY     ZETA     UMEGA        CPS                  REAL        IMAGINARY     ZETA     OMEGA       CPS
 1      -2.51           -0.388       0.988   2.54       0.404         2      -0.490          1.99        0.239   2.05       0.326
```

APPENDIX A: QR EIGENVALUE CALCULATION PROCESS

An elementary method for extracting roots of a matrix of polynomials is first to expand the determinant into a single polynomial.

$$\|[F]\| = Q(s) \quad \text{where } Q(s) \text{ is a polynomial} \qquad (A - 1)$$

If the roots of $Q(s) = 0$ were extracted, they would be identical to the roots of $\|[F]\| = 0$. Although this method is cumbersome and of little practical use, insight may be gained by viewing the problem in this manner since the roots (or eigenvalues) are so instrumental in QR calculations. Specifically, the extent to which the polynomial, $Q(s)$, is modified by row and column operations, can be viewed in terms of the rules applying to row and column operations of determinants. These determinant rules will be used to perform all possible preliminary reductions of the [F] matrix.

After completion of the reductions, the remaining matrix will be transformed to eigenvalue form. The two principal elements of this operation will be to first change the variables so that the matrix has no derivative higher than first order. The first order matrix will then be normalized (diagonalized) to produce the desired eigenvalue problem.

A. PRELIMINARY REDUCTIONS

The matrix will be reduced in size by use of the following properties of determinants.

1. SINGULAR MATRICES

If any row (or column) is zero, the matrix is singular, and its determinant is zero.

$$\begin{vmatrix} 11. & 12. & 0 & 14. \\ 21. & 22. & 0 & 24. \\ 31. & 32. & 0 & 34. \\ 41. & 42. & 0 & 44. \end{vmatrix} = 0 \qquad (A - 2)$$

2. EXPANSION BY CO-FACTORS OF A SPARSE ROW OR COLUMN

If any row (or column) is zero except for one element, the determinant may be reduced by one order. The row and column corresponding to the element are removed, and the element is written outside of the determinant as a factor. If the element is a polynomial, the roots of the polynomial are extracted. The determinant is multiplied by $(-1)^{i+j}$, where i and j are the row and column subscripts of the non-zero element of the row or column which is being removed.

$$\begin{vmatrix} 11. & 12. & 0 & 14. \\ 21. & 22. & 23. & 24. \\ 31. & 32. & 0 & 34. \\ 41. & 42. & 0 & 44. \end{vmatrix} = -23. \begin{vmatrix} 11. & 12. & 14. \\ 31. & 32. & 34. \\ 41. & 42. & 44. \end{vmatrix} \qquad (A - 3)$$

3. SCALAR MULTIPLICATION

If any row (or column) is multiplied by a factor, the determinant is multiplied by that same factor.

$$\begin{vmatrix} 11. & 12. & 13. & 14. \\ 21. & 22. & 23. & 24 \\ k31. & k32. & k33. & k34. \\ 41. & 42. & 43. & 44. \end{vmatrix} = k \begin{vmatrix} 11. & 12. & 13. & 14. \\ 21. & 22. & 23. & 24. \\ 31. & 32. & 33. & 34. \\ 41. & 42. & 43. & 44. \end{vmatrix} \tag{A - 4}$$

4. INTERCHANGING ROWS OR COLUMNS

If any two rows (or two columns) are interchanged, the determinant changes sign.

$$\begin{vmatrix} 11. & 12. & 13. & 14. \\ 41. & 42. & 43. & 44. \\ 31. & 32. & 33. & 34. \\ 21. & 22. & 23. & 24. \end{vmatrix} = - \begin{vmatrix} 11. & 12. & 13. & 14. \\ 21. & 22. & 23. & 24. \\ 31. & 32. & 33. & 34. \\ 41. & 42. & 43. & 44. \end{vmatrix} \tag{A - 5}$$

5. ADDITION OF ROWS OR COLUMNS

If any row (or column) has another row (or column) added to it, the determinant is unchanged. The determinant is unchanged even if the added row has been multiplied by a constant.

$$\begin{vmatrix} 11. & 12. + k14. & 13. & 14. \\ 21. & 22. + k24. & 23. & 24. \\ 31. & 32. + k34. & 33. & 34. \\ 41. & 42. + k44. & 43. & 44. \end{vmatrix} = \begin{vmatrix} 11. & 12. & 13 & 14. \\ 21. & 22. & 23. & 24. \\ 31. & 32. & 33. & 34. \\ 41. & 42. & 43. & 44. \end{vmatrix} \tag{A - 6}$$

6. GUASSIAN REDUCTION

The last operation leads to Guassian reduction. Multipliers are selected such that a row (or column) is forced to be zero except for one element. For example, the first column of the matrix below will be cleared of terms, except for the 3, 1 element, by performing algebraic operations on its rows.

$$\|F\| = \begin{vmatrix} 11. & 12. & 13. & 14. \\ 21. & 22. & 23. & 24. \\ 31. & 32. & 33. & 34. \\ 41. & 42. & 43. & 44. \end{vmatrix} \tag{A - 7}$$

Add -11/31 times row 3 to row 1.

Add -21/31 times row 3 to row 2.

Add -41/31 times row 3 to row 4.

$$\|[F]\| = \begin{vmatrix} 11 - (11/31)31 & 12 - (11/31)32 & 13 - (11/31)33 & 14 - (11/31)34 \\ 21 - (21/31)31 & 22 - (21/31)32 & 23 - (21/31)33 & 24 - (21/31)34 \\ 31 & 32 & 33 & 34 \\ 41 - (41/31)31 & 42 - (41/31)32 & 43 - (41/31)33 & 44 - (41/31)34 \end{vmatrix} \qquad (A - 8)$$

$$\|[F]\| = \begin{vmatrix} 0 & .646 & 1.292 & 1.937 \\ 0 & .323 & .646 & .968 \\ 31 & 32 & 33 & 34 \\ 0 & -.320 & -.643 & -.965 \end{vmatrix} \qquad (A - 9)$$

The foregoing operation allows reduction to a 3 x 3 determinant by writing element 3, 1 outside the determinant as a scalar multiplier and eliminating the first column and third row. However, if column 1 had contained polynomials, those polynomials would have been scattered throughout the array. Except for special situations, operations with polynomial elements are of no use.

SYSTEMATIC REDUCTION OF MATRICES

By applying the above operations, dramatic reductions in the matrix size may be possible. Also, removable factors sometimes cause difficulty to matrix rooting schemes if those factors are not removed. Thus, before transforming an eigenvalue problem, all possible reductions should be made to the [F] matrix. The reductions are separated into three cases:

Case 1 – Row or Column Zero Except for One Element

Case 2 – A Row or Column With Only One Power of s (including zero)

Case 3 – A Row or Column With a Common Polynomial Factor

The reduction process is illustrated by example. The following matrix is assumed:

$$Q(s) = \begin{vmatrix} 2s^2 + 3s + 9 & -5s^2 + 6 & 0 & 0 & -4s^2 & 0 \\ -2s + 4 & 4s + 4 & 0 & 4s + 4 & 0 & -4 \\ 0 & -20 & s + 20 & 0 & s + 10 & 0 \\ 0 & 6 & 0 & -2s - 2 & 0 & 2 \\ 5s & 0 & 0 & 0 & 10s & 0 \\ 3s & 0 & 0 & 0 & 0 & -s - 15 \end{vmatrix} \qquad (A - 10)$$

CASE 1: ROW OR COLUMN ZERO EXCEPT FOR ONE ELEMENT

Search all rows and columns for a row or column with only one* non-zero element. The 3, 3 element satisfies that requirement. The determinant is reduced by removing row 3 and column 3. The factor s + 20 is brought outside of the determinant. The sign of the determinant is still positive because $(-1)^{i+j} = (-1)^{3+3} = (-1)^6 = +1$.

$$Q(s) = (s + 20) \begin{bmatrix} 2s^2 + 3s + 9 & -5s^2 + 6 & 0 & -4s^2 & 0 \\ -2s + 4 & 4s + 4 & 4s + 4 & 0 & -4 \\ 0 & 6 & -2s - 2 & 0 & 2 \\ 5s & 0 & 0 & 10s & 0 \\ 3s & 0 & 0 & 0 & -s - 15 \end{bmatrix} \qquad (A-11)$$

CASE 2: A REDUCIBLE ROW OR COLUMN WITH LIKE POWERS OF s

If a row or column has only one power of s, the power of s can be removed as a factor. The row or column is then constant and can be operated upon by Guassian elimination. The determinant is re-examined for a single element in a row or column. No such situation exists for the example. **Next**, all rows and columns are examined for a row or column with only one power of s. Row 4 satisfies that requirement. Row 4 is reduced by subtracting -1/2 times column 4 from column 1:

$$Q(s) = (s + 20) \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 & 0 & -4s^2 & 0 \\ -2s + 4 & 4s + 4 & 4s + 4 & 0 & -4 \\ 0 & 6 & -2s - 2 & 0 & 2 \\ 0 & 0 & 0 & 10s & 0 \\ 3s & 0 & 0 & 0 & -s - 15 \end{bmatrix} \qquad (A-12)$$

Thus, the determinant now satisfies the Case 1 conditions: Row 4 is zero except for the 4, 4 element. Factoring +10s (positive because $(-1)^{4,4} = +1$) and dropping Row 4 and column 4:

$$Q(s) = (10s)(s + 20) \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 & 0 & 0 \\ -2s + 4 & 4s + 4 & 4s + 4 & -4 \\ 0 & 6 & -2s - 2 & 2 \\ 3S & 0 & 0 & -s - 15 \end{bmatrix} \qquad (A-13)$$

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*If a column or row is found to be all zero, the matrix is singular. The procedure can go no further. No roots can be found.

CASE 3: A REDUCIBLE ROW OR COLUMN WITH COMMON POLYNOMIAL FACTOR

The matrix is again examined for the Case 1 conditions, but that situation does not exist. Case 2 conditions are also absent. However, the third column satisfies Case 3 because the common factor k(s + 1) is present (k an arbitrary constant). Column 3 is reduced by adding 1/2 times row 2 to row 3:

$$Q(s) = (10s)(s + 20) \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 & 0 & 0 \\ -2s + 4 & 4s + 4 & 4s + 4 & -4 \\ -s + 2 & 2s + 8 & 0 & 0 \\ 3s & 0 & 0 & -s - 15 \end{bmatrix} \qquad (A - 14)$$

The Case 1 conditions are thereby provided since element 2, 3 is the only non-zero element in column 3. Factoring $(-1)^{2+3}(4s + 4) = (-1)^5(4s + 4) = -4(s + 1)$ and removing row 2 and column 3:

$$Q(s) = -4(s + 1)(10s)(s + 20) \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 & 0 \\ -s + 2 & 2s + 8 & 0 \\ 3s & 0 & -s - 15 \end{bmatrix} \qquad (A - 15)$$

The last reduction (Case 3) created another Case 1 situation with column 3 being zero except for element 3, 3. Thus, the point is made for re-checking the determinant for all cases on each cycle of the reduction process. Factoring -(s + 15) and removing row 3 and column 3.

$$Q(s) = 40(s + 15)(s + 1)(s)(s + 20) \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 \\ -s + 2 & 2s + 8 \end{bmatrix} \qquad (A - 16)$$

Since the remaining 2 x 2 matrix does not satisfy any of the three cases for reduction, further application of the reduction conditions is futile. The 2 x 2 determinant will once again be treated as a matrix, and its roots will be found by forming a eigenvalue problem. The total set of roots of the original matrix will consist of the 2 x 2 matrix eigenvalues and the roots of the removed factors.

Note that the processes of Cases 1 and 2 are specializations of Case 3. Also, Case 1 is a specialization of Case 2. However, Case 3 seldom occurs in practice. Case 2 occurs much less often than Case 1. By appointing the cases in order, a computing program can be made to execute more efficiently.

B. FORMATION OF THE EIGENVALUE PROBLEM

The remaining work consists of separating the problem into a constant matrix and a first order matrix, $\left[A_0\right] + s\left[A_1\right] \{y\} = \{0\}$. The $\left[A_1\right]$ matrix will be normalized and the resulting constant matrix will be moved to the right side of the equation, $s\{y\} = [A]\{y\}$.

1. TRANSFORMATION TO FIRST ORDER FORM

A systematic approach is used to put the problem into first order form. This approach will correspond to the computer algorithm. Two singularly dimensioned arrays will be used to sort the matrix columns into their respective locations in $\left[A_0\right]$ and $\left[A_1\right]$.

IORD(i), i = 1, m - - an array whose elements are equal to the highest power of s in each column. If a column is constant, IORD-1 for that column. The constant, m, is equal to the size of the [F] matrix.

ID(i), i = 1, md - - an identification array to keep track of the columns, segregated according to the power of s. The elements of ID are equal to the column number plus 1000 times the power of s for that column. The constant md is equal to the matrix size (m) times the highest power of s plus one (md).

The previous example will be used to illustrate.

$$\begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 \\ -s + 2 & 2s + 8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \text{(A - 16)}$$

where x_1 and x_2 are dummy variables.

Values of m and md: m = 2 (a 2 x 2 matrix)
md = 3 (highest power of s plus 1)

The IORD array: IORD = (2, 2), i.e. both columns have at least one second order coefficient.

The ID array: ID = (1, 2, 1001, 1002, 2001, 2002), i.e. The columns are ordered in ascending powers of s.

Thus, the IORD and ID arrays are associated with the matrix as follows:

$$\text{IORD} = \begin{array}{c} \begin{array}{cc} \quad 2 \quad & \quad\quad 2 \end{array} \\ \begin{bmatrix} 4s^2 + 3s + 9 & -5s^2 + 6 \\ -s + 2 & 2s + 8 \end{bmatrix} \end{array} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\text{ID} = \left[\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 9 & 6 \\ 2 & 8 \end{bmatrix} \end{array} + s \begin{array}{c} \begin{array}{cc} 1001 & 1002 \end{array} \\ \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \end{array} + s^2 \begin{array}{c} \begin{array}{cc} 2001 & 2002 \end{array} \\ \begin{bmatrix} 4 & -5 \\ 0 & 0 \end{bmatrix} \end{array} \right] \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \text{(A - 17)}$$

Next, the sum of the IORD array is formed.

$$\sum_{i=1}^{2} \text{IORD}\,(i) = k = 4 \left(\text{the size of } \left[A_0\right] \text{ and } \left[A_1\right]^*\right)$$

Hence, k is the apparent size of the state variable problem.

The $\left[A_0\right]$ matrix is formed from columns of the original matrix [F]. The hierarcy of column storage requires the column order to be unmodified from the column order of [F]. Within each column, the separate columns for each power of s must be arranged in decending order starting with the IORD-1 power of s.

$$A_0 = \begin{array}{c} \text{ID} = \begin{array}{cccc} 1001 & 1 & 1002 & 2 \end{array} \\ \left[\begin{array}{cccc} 3 & 9 & 0 & 6 \\ -1 & 2 & 2 & 8 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \qquad \text{(A - 18)}$$

The identification array (ID) is a statement of the hierarcy; that is, first column and first power of s, first column and zero power of s, etc.

The $\left[A_1\right]$ matrix is formed from the highest power of s columns. If a column is constant, the s^1 terms are used. The first column highest power of s is stored into the first column of $\left[A_1\right]$. Then IORD-1 columns are skipped. This process continues until all of the highest power of s columns of [F] have been stored into $\left[A_1\right]$.

$$A_1 = \begin{array}{c} \text{ID} = \begin{array}{cccc} 2001 & & 2002 & \end{array} \\ \left[\begin{array}{cccc} 4 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \qquad \text{(A - 19)}$$

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*By inspection, the fourth order size is faulty because the determinant of the [F] matrix is third order. However, the normalization process will rectify this shortcoming. The order obtained by inspecting the columns will be referred to as the apparent order.

The state variables are determined from the first n values of the ID array For the example ID = (1001, 1, 1002, 2) is interpreted as $\left(\dot{x}_1, x_1, \dot{x}_2, x_2 \right)$.

The first order problem can now be assembled by adding the additional equations relating the second order and higher derivatives.

$$s \left[\frac{d^{j-2}}{dt^{j-2}} (x_i) \right] - \frac{d^{j-1}}{dt^{j-1}} (x_i) = 0 \qquad \text{(A - 20)}$$

i = 1, m (the original variable numbers)

j = 2, IORD(i) (If IORD(i) = 1, the equation does not exist for that i.)

For the same problem,

$$s \left[\frac{d^0}{dt^0} (x_1) \right] - \frac{d^1}{dt^1} (x_1) = 0; \quad sx_1 - \dot{x}_1 = 0$$

$$s \left[\frac{dt^0}{dt^0} (x_2) \right] - \frac{dt^1}{dt^1} (x_2) = 0; \quad sx_2 - \dot{x}_2 = 0 \qquad \text{(A - 21)}$$

$$\left[\begin{bmatrix} 3 & 9 & 0 & 6 \\ -1 & 2 & 2 & 8 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 4 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right] \begin{Bmatrix} \dot{x}_1 \\ x_1 \\ \dot{x}_2 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(A - 22)}$$

or,

$$\left[[A_o] + s \, [A_1] \right] \{y\} = \{0\}$$

A change in sign may have occurred in the transformation to first order form. The sign change is $(-1)^{cp}$. The integer, cp, equals the number of column permutations that would be required if the columns of $[A_1]$ which contain the original columns of [F] are packed to the left. For the sample problem the $[A_1]$ matrix is packed to the left if columns 2 and 3 are switched. Thus, cp = 1 and the removed factor changes sign.

Removed Factor = $-40(s + 15)(s + 1)(s)(s + 20)$

2. NORMALIZING THE FIRST ORDER MATRIX

If the first order matrix $\left[A_1\right]$, is non-singular, the eigenvalue problem would be

$$s\,\{y\} = -\left[A_1\right]^{-1}\left[A_0\right]\{y\} = \left[A\right]\{y\} \qquad \text{(A - 23)}$$

Since, in general, $\left[A_1\right]$ is singular (e.g. the same problem), special steps must be pursued.

The first task is to put the matrix into normal form by using Guassian elimination by row operations. That process is continued until no pivot can be found. For the sample problem, $\left[A_1\right]$ is searched for the largest magnitude value - the 1, 3 element = -5. Row 1 is divided by -5. Guassian elimination along column 3 will produce no change in the matrix since all other elements in column 3 are zero. Exchanging* columns 1 and 3 leads to the following form:

$$\left[\begin{bmatrix} 0 & -1.8 & -.6 & -1.2 \\ 2 & 2 & -1 & 8 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & -.8 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}_2 \\ x_1 \\ \dot{x}_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(A-24)}$$

To keep the determinant invariant, the row 1 division by -5 must be compensated by multiplying the removed factory by -5. The sign of the factor must change to account for the column swap.

Removed Factor = -(-5) (-40) (s + 15) (s + 1) (s) (s + 20)

= -200(s + 15) (s + 1) (s) (s + 20)

The matrix is again searched for a pivot, but the search must exclude row 1 and column 1. The pivotal element in 3, 2 is selected. The pivot is already equal to 1 and no division is required. Guassian reduction is not required because column 2 is zero except for the pivot. Rows 2 and 3 are switched and the removed factor changes sign.

$$\left[\begin{bmatrix} 0 & -1.8 & -.6 & -1.2 \\ 0 & 0 & -1 & 0 \\ 2 & 2 & -1 & 8 \\ -1 & 0 & 0 & 0 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & -.8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}_2 \\ x_1 \\ \dot{x}_1 \\ x_2 \end{Bmatrix} = \{0\} \qquad \text{(A - 25)}$$

Removed Factor = +200(s + 15)(s + 1)(s)(s + 20)

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*Exchanging rows and columns is used for illustration only. The computer program keeps track of the pivotal locations and performs the switching after all normalization is complete.

The pivot search now must exclude rows 1 and 2 and columns 1 and 2. The 4, 4 element is selected. Division and Guassian elimination are not required. Rows 3 and 4 are switched, and the removed factor changes sign. Columns 3 and 4 are switched and the removed factor changes sign again.

$$\left[\begin{bmatrix} 0 & -1.8 & -1.2 & -.6 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 2 & 2 & 8 & -1 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & 0 & -.8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}_2 \\ x_1 \\ x_2 \\ \dot{x}_1 \end{Bmatrix} = \{0\} \qquad \text{(A - 26)}$$

Removed Factor = + 200(s + 15)(s + 1)(s)(s + 20)

The pivot search must exclude rows and columns 1, 2 and 3. Thus, the 4, 4 element is the only candidate and that element is zero.

The matrix equation is in the following form:

$$\left[\left[\begin{array}{c|c} \alpha_{11} & \alpha_{12} \\ \hline \alpha_{21} & \alpha_{22} \end{array}\right] + s\left[\begin{array}{c|c} \beta_{11} & \beta_{12} \\ \hline \beta_{21} & \beta_{22} \end{array}\right]\right]\left\{\begin{array}{c} z_2 \\ \hline z_2 \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline 0 \end{array}\right\} \qquad \text{(A - 27)}$$

where:

α_{ij} and β_{ij} are matrix partitions

β_{11} = I, and identity partition

β_{12} is non-zero in general

β_{21} is zero due to the Guassian reductions

β_{22} is zero because no pivot could be found

z_1 and z_2 are the corresponding variables.

$$\left[\left[\begin{array}{c|c} \alpha_{11} & \alpha_{12} \\ \hline \alpha_{21} & \alpha_{22} \end{array}\right] + s\left[\begin{array}{c|c} I & \beta_{12} \\ \hline 0 & 0 \end{array}\right]\right]\left\{\begin{array}{c} z_1 \\ \hline z_2 \end{array}\right\} = \{0\} \qquad \text{(A - 28)}$$

Guassian elimination by column operations will be used to force to zero.

For the sample problem, the Guassian elimination consists of adding .8 times column 1 to column 4.

$$\left[\begin{bmatrix} 0 & -1.8 & -1.2 & -.6 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & -.8 \\ 2 & 2 & 8 & .6 \end{bmatrix} + s \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right] \begin{Bmatrix} \dot{x}_2' \\ x_1 \\ x_2 \\ \dot{x}_1 \end{Bmatrix} = \{0\} \qquad \text{(A - 29)}$$

The general form is now,

$$\left[\left[\begin{array}{c|c} \alpha_{11} & \alpha_{12}' \\ \hline \alpha_{21} & \alpha_{22}' \end{array}\right] + s \left[\begin{array}{c|c} I & 0 \\ \hline 0 & 0 \end{array}\right]\right] \begin{Bmatrix} z_1' \\ z_2 \end{Bmatrix} = \{0\} \qquad \text{(A - 30)}$$

Guassian elimination of the columns of $\left[A_0\right]$ will be used to eliminate the columns which comprise α_{12}' and α_{22}'.

The pivot search must extend to all elements of α'_{12} and α'_{22}. For the sample case, the pivot is -1 in row 2. Row 2 is divided by -1.

$$\left[\begin{bmatrix} 0 & -1.8 & -1.2 & -.6 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -.8 \\ 2 & 2 & 8 & .6 \end{bmatrix} + s \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right] \begin{Bmatrix} \dot{x}'_2 \\ x_1 \\ x_2 \\ \dot{x}_1 \end{Bmatrix} = \{0\} \qquad \text{(A - 31)}$$

Removed factor = -200(s + 15)(s + 1)(s)(s + 20)

Then, .6 times row 2 is added to row 1,
.8 times row 2 is added to row 3, and
-.6 times row 2 is added to row 4.

$$\left[\begin{bmatrix} 0 & -1.8 & -1.2 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 2 & 2 & 8 & 0 \end{bmatrix} + s \begin{bmatrix} 1 & -.6 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -.8 & 1 & 0 \\ 0 & .6 & 0 & 0 \end{bmatrix}\right] \begin{Bmatrix} \dot{x}'_2 \\ x_1 \\ x_2 \\ \dot{x}_1 \end{Bmatrix} = \{0\} \qquad \text{(A - 32)}$$

The Guassian reduction forces the un-normalized columns to be zero except for one element. The matrix can be reduced and the normalization process resumed.

$$\left[\begin{bmatrix} 0 & -1.8 & -1.2 \\ -1 & 0 & 0 \\ 2 & 2 & 8 \end{bmatrix} + s\begin{bmatrix} 1 & -.6 & 0 \\ 0 & -.8 & 1 \\ 0 & .6 & 0 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}'_2 \\ x_1 \\ x_2 \end{Bmatrix} = \{0\} \qquad (A - 33)$$

Removed Factor = -200(s + 15)(s + 1)(s)(s + 20)

If 1,1 element is selected as a pivot, no division or Guassian elimination is required for column 1.

The 2,3 element is already normalized to 1 and column 3 requires no Guassian elimination. Swapping columns 2 and 3,

$$\left[\begin{bmatrix} 0 & -1.2 & -1.8 \\ -1 & 0 & 0 \\ 2 & 8 & 2 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & -.6 \\ 0 & 1 & -.8 \\ 0 & 0 & .6 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}'_2 \\ x_2 \\ x_1 \end{Bmatrix} = \{0\} \qquad (A - 34)$$

Removed Factor = 200(s + 15)(s + 1)(s)(s + 20)

The pivot search is restricted to row 3 and column 3. Hence, the pivot is .6. Dividing row 3 by .6,

$$\left[\begin{bmatrix} 0 & -1.2 & -1.8 \\ -1 & 0 & 0 \\ 3.33 & 13.33 & 3.33 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & -.6 \\ 0 & 1 & -.8 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{Bmatrix} \dot{x}'_2 \\ x_2 \\ x_1 \end{Bmatrix} = \{0\} \qquad (A - 35)$$

Removed Factor = 120 (s + 15)(s + 1)(s)(s + 20)
Add .8 times row 3 to row 2, and
add .6 times row 3 to row 1.

$$\left[\begin{bmatrix} 2 & 6.8 & 2 \\ 1.67 & 10.67 & 2.67 \\ 3.33 & 13.33 & 3.33 \end{bmatrix} + s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{Bmatrix} x'_2 \\ x_2 \\ x_1 \end{Bmatrix} = \{0\} \qquad (A - 36)$$

The eigenvalue problem is formed by moving the constant matrix to the right side of the equation.

$$s\begin{Bmatrix} \dot{x}'_2 \\ x_2 \\ x_1 \end{Bmatrix} = \begin{bmatrix} -2 & -6.8 & -.2 \\ -1.67 & -10.67 & -2.67 \\ -3.33 & -13.33 & -3.33 \end{bmatrix} \begin{Bmatrix} \dot{x}'_2 \\ x_2 \\ x_1 \end{Bmatrix} \qquad (A-37)$$

or,

$$\{\dot{x}\} = [A]\,\{x\} \qquad (A-38)$$

The roots of **[A]** may be found by using the QR algorithm (see ref. 3).

$s = 15.$

$s = -.488 \pm j1.05$

The complete set of roots is formed by combining the above roots with roots of the removed factor. The seven roots of the sample problem follow.

$s = -20$

$= -15$

$= -15$

$= -.488 + j1.05$

$= -1$

$= 0$

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APPENDIX B: QR STATE SPACE FORMULATION

This section describes the procedure QR uses for transforming linear, constant coefficient, equations state variable form.

$$\underset{m\times m}{[F]}\ \{q\} = \{U\} \Rightarrow \begin{cases} \{x\} = \underset{n\times n}{[A]}\ \{x\} + \underset{n\times m}{[B]}\ \{U\} \\ \\ \{y\} = \underset{k\times n}{[C]}\ \{x\} + \underset{k\times m}{[D]}\ \{U\} \quad \Rightarrow \{ID\} \end{cases} \tag{B - 1}$$

where:

[F] differential equations matrix in terms of differential operator.

{q} vector of variables

{U} vector of inputs

[A] state variable coefficient matrix

[B] state variable input matrix

[C] output matrix for state variables

[D] output matrix for inputs

{x} state variables

{y} output variables (composed of all of the q's and possibly some of the derivatives of the q's

{ID} an identification array which relates to y's to the q's and derivatives of the q's

Thus, given [F] the state variable problem is transformed to [A], [B], [C], [D] and {ID}

In appendix A, reductions were performed on [F] prior to transformation to a first order matrix equation. Those reductions are not advisable for the general case because identification of the state variables becomes difficult. Therefore, the procedure herein will transform directly to first order form and then normalize. The preliminary reduction process will be bypassed.

1. TRANSFORMATION TO FIRST ORDER FORM

The method to transform to first order form used in Appendix A also applies to the general state variable problem. However, the right-hand side {U}, must be retained. The sample problem is repeated below and then put into first order form

$$\begin{bmatrix} 2s^2+3s+9 & -5s^2+6 & 0 & 0 & -4s^2 & 0 \\ -2s+4 & 4s+4 & 0 & 4s+4 & 0 & -4 \\ 0 & -20 & s+20 & 0 & s+10 & 0 \\ 0 & 6 & 0 & -2s-2 & 0 & 2 \\ 5s & 0 & 0 & 0 & 10s & 0 \\ 3s & 0 & 0 & 0 & 0 & -s-15 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} \qquad \text{(B - 2)}$$

$$\Downarrow$$

$$\begin{array}{l} \text{ID} = \begin{array}{ccccccccc} 1001 & 1 & 1002 & 2 & 3 & 4 & 1005 & 5 & 6 \end{array} \\ \begin{bmatrix} 3 & 9 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ -2 & 4 & 4 & 4 & 0 & 4 & 0 & 0 & -4 \\ 0 & 0 & 0 & -20 & 20 & 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 6 & 0 & -2 & 0 & 0 & 2 \\ 5 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -15 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

$$+\, s \begin{bmatrix} 2 & 0 & -5 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \end{Bmatrix} = \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(B - 3)}$$

where:

$$\left(y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6,\ y_7,\ y_8,\ y_9 \right)$$
$$= \left(\dot{q}_1,\ q_1,\ \dot{q}_2,\ q_2,\ q_3,\ q_4,\ \dot{q}_5,\ q_5,\ q_6 \right)$$

The nine apparent state variables are identified by the first nine elements of the ID vector. The form of the problem is now,

$$\left[[A_0] + s\,[A_1] \right] \{y\} = \left\{ \frac{U}{0} \right\} \tag{B - 4}$$

B. NORMALIZING THE FIRST ORDER MATRIX

For Option 1, Guassian eliminations were used to normalize the matrix. Singularities of the first order matrix were removed by performing Guassian elimination on the constant matrix. The normalization cycle will be identical to Option 1 except bookkeeping must be maintained with regard to the effect the operations have on the input and output variables. This effect can be visualized by expressing Guassian elimination as a matrix multiplication. For the case of Guassian elimination by row operations,

$$[V_\ell]\ [A] = [A']$$

where $[V_\ell]$ is a matrix which normalizes row l and zeros all elements in column l except for the l, l element.

The $[V_\ell]$ matrix is an identity matrix except for the l column.

$$[V_\ell] = \begin{bmatrix} 1 & & & V_{1\ell} & & & \\ & 1 & & V_{2\ell} & & & \\ & & \ddots & \vdots & & & \\ & & & V_{\ell\ell} & & & \\ & & & \vdots & \ddots & & \\ & & & V_{n\ell} & & & 1 \end{bmatrix} \tag{B - 5}$$

The $v_{\ell\ell}$ element is equal to the constant required to normalize the $A_{\ell\ell}$ element to 1. The other v's perform the row operations.

v_{i1} $(i \neq 1)$ adds $v_{i\ell}$ times row 1 to row i.

Thus, one Guassian elimination step by row operations can be expressed by a premultiplication by $[V_\ell]$

$$[V_\ell]\Big[[A_0] + s[A]\Big]\{y\} = [V_\ell]\{U\} = \{U'\} \qquad \text{(B - 6)}$$

The purpose of expressing the operation in the above manner was to illustrate the effect on $\{y\}$ and $\{U\}$.

$\{y\}$ was undisturbed

$\{U\}$ was transformed to $\{U'\} = [V_\ell]\{U\}$

In a similar manner a $[T_\ell]$ matrix can be devised to perform Guassian elimination by column operations.

$$[T_\ell] = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & \cdot & & & & & \\ & & & \cdot & & & & \\ & & & & \cdot & & & \\ t_{\ell 1} & t_{\ell 2} & \cdots & & t_{\ell\ell} & \cdots & & t_{\ell n} \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & & \cdot \\ & & & & & & & 1 \end{bmatrix} \qquad \text{(B - 7)}$$

$t_{\ell\ell}$ normalizes $A_{\ell\ell}$ to 1

t_{1j} $(j \neq 1)$ adds $t_{\ell j}$ times column 1 to column j.

The operation as pertains to the first order problem,

$$\Big[[A_0] + s[A]\Big][T_\ell]\{y'\} = \{U\} \qquad \text{(B - 8)}$$

$\{y\}$ is transformed to $\{y'\} = [T_1]\{y\}$

$\{U\}$ is unchanged

If interchanges of row and columns are required, operational matrices can also be devised.

$$[V_{k\ell}] = \left[\begin{array}{c|ccccccccccc} & 1 & & & & & & & & & & \\ & & \ddots & & & & & & & & & \\ & & & 1 & & & & & & & & \\ k & & & & 0 & & & & 1 & & & \\ & & & & & 1 & & & & & & \\ & & & & & & \ddots & & & & & \\ & & & & & & & 1 & & & & \\ \ell & & & & 1 & & & & 0 & & & \\ & & & & & & & & & 1 & & \\ & & & & & & & & & & \ddots & \\ & & & & & & & & & & & 1 \end{array}\right] \tag{B - 9}$$

$$[V_{k\ell}]\ [A] = [A']$$

interchanges
rows k and ℓ of [A]

$$[T_{k\ell}] = \left[\begin{array}{ccccccccccc} & & & k & & & & \ell & & & \\ \hline 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 0 & & & & 1 & & & \\ & & & & 1 & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 1 & & & & \\ & & & 1 & & & & 0 & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & \ddots & \\ & & & & & & & & & & 1 \end{array}\right] \tag{B - 10}$$

$$[A]\ [T_{k\ell}] = [A']$$

interchanges
columns k and ℓ of [A].

All of the row and column operations can be grouped together as,

$$\left[V_\gamma\right] \cdot \cdot \cdot \left[V_2\right]\left[V_1\right]\left[\left[A_0\right] + s\left[A_1\right]\right]\left[T_1\right]\left[T_2\right] \cdot \cdot \cdot \left[T_\lambda\right]\{y\} = \{U\}$$

$$\{y'\} = \left[T_1\right]\left[T_2\right] \cdot \cdot \cdot \cdot \cdot \left[T_\lambda\right]\{y\}$$

$$\{U'\} = \left[V_\gamma\right] \cdot \cdot \cdot \cdot \cdot \left[V_2\right]\left[V_1\right]\{U\} \qquad \text{(B-11)}$$

where the [T] or [V] matrices perform either Guassian elimination, or row or column interchanges.

The actual computer operations do not require making [T] and [V] matrices and carrying out the matrix multiplications. The first order matrix equation will be restated as,

$$\left[\left[A_0\right] + s\left[A_1\right]\right][T]\{y\} = [V]\{U\} \qquad \text{(B-12)}$$

where [T] is an identity matrix comfortable with $\{y\}$, and [V] is an identity matrix of the same order as [T] but with only enough columns to be comfortable with $\{U\}$.

For the sample problem, [T] is a ninth order identity matrix with columns 7, 8 and 9 missing.

The strategy used during the normalization cycle will be as follows:

- If Guassian elimination by row operations is performed on $\left[A_0\right]$ and $\left[A_1\right]$, the same operations will be performed on [V].
- If Guassian elimination by column operations is performed on $\left[A_0\right]$ and $\left[A_1\right]$, the same operations will be performed on [T].
- If two rows of $\left[A_0\right]$ and $\left[A_1\right]$ are switched, the same rows will be switched in [V].
- If two rows of $\left[A_0\right]$ and $\left[A_1\right]$ are switched, the same rows will be switched in [T].

In summary, row operations will act upon [V], and the column operations will act upon [T].

CONTENTS

Page

1.0 SUMMARY 1 1/A9
2.0 INTRODUCTION 3 1/A11
3.0 SYMBOLS AND ABBREVIATIONS 4 1/A12
4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION 8 1/B2
  4.1 The Basic Polynomial Equation 8 1/B2
  4.2 Open-Loop and Closed-Loop Systems 10 1/B4
  4.3 Transfer Function Formation 13 1/B7
  4.4 Eigenvalue Calculation 15 1/B9
  4.5 State Space Formulation 16 1/B10
  4.6 Frequency Response Calculation 17 1/B11
  4.7 Power Spectral Density Analysis 18 1/B12
  4.8 Time Response Calculations 21 1/C1
    4.8.1 Time Response by Method of Residues 21 1/C1
    4.8.2 Time Response With Tau 23 1/C3
    4.8.3 Time Response for Forcing Functions that Do Not Have Simple Laplace Transforms 23 1/C3
  4.9 Unsteady Aerodynamic Options 24 1/C4
    4.9.1 Frequency Response Analysis 25 1/C5
    4.9.2 Flutter Analysis 27 1/C7
5.0 PROGRAM STRUCTURE AND DESCRIPTION 29 1/C9
  5.1 QR Program Control and Eigenvalue Rooting 29 1/C9
  5.2 Time Response Calculations 29 1/C9
  5.3 Frequency Response and PSD Calculations 31 1/C12
  5.4 Root Locus and V-g Plotting 31 1/C12
  5.5 Unsteady Airforce Frequency Response and Flutter Calculations 31 1/C12
6.0 COMPUTER PROGRAM USAGE 32 1/C13
  6.1 Control Cards 32 1/C13
  6.2 Resource Estimates 33 1/C13
  6.3 Card Input Data 34 1/D1
    6.3.1 Case Initialization 35 1/D2
    6.3.2 Matrix Element Operations 36 1/D3
    6.3.3 Roots of Matrix and Root Locus 57 1/E10
    6.3.4 Transfer Function Formation 69 1/F8
    6.3.5 Time Response 73 1/F12
    6.3.6 Frequency Response 79 1/G4
    6.3.7 Unsteady Aerodynamics Options 84 1/G9
    6.3.8 Cycling Control Cards and Labeling 94 2/A7
    6.3.9 Input Card Summary 98 2/A11

CONTENTS (CONCLUDED)

Page

6.4 Examples of Required Card Sets for Some Typical Problems 107 2/C1
6.5 Output Data 109 2/C3
    6.5.1 Printed Output Data 109 2/C3
    6.5.2 File Output 109 2/C3
6.6 Restructions 115 2/C9
6.7 Diagnostics 115 2/C9
    6.7.1 READTP Error Codes 118 2/C12
    6.7.2 WRTFTP Error Codes 119 2/C13
6.8 Use of User Supplies Subroutines 119 2/C13
7.0 SAMPLE PROBLEMS 127 2/D7
7.1 Eigenvalue Calculation 12~~8~~7 2/D7
7.2 Time Response Calculation 128 2/D8
7.3 Frequency Response and Power Spectral Density Calculation 131 2/D11
7.4 Root Locus Calculation 133 2/D13
7.5 Sample Problem Output 135 2/E1
Appendix A: QR Eigenvalue Calculation Process 147 2/F9
Appendix B: QR State Space Formulation 161 2/G9
REFERENCES 175 3/A11

The sample problem is too cumbersome for presentation of all the steps. Therefore, only the end result of the normalization process is shown.

$$\left[\begin{bmatrix} 2 & 0 & 0 & 0 & .2 & -6.8 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -3 & 0 & 0 & 0 \\ 1.67 & 0 & 20 & 0 & 1.67 & -13.33 & 10 & 0 & 0 \\ 10 & 0 & 0 & 15 & 10 & 40 & 0 & 0 & 0 \\ 3.33 & 0 & 0 & 0 & 3.33 & 13.33 & 0 & 0 & 0 \\ 1.67 & 0 & 0 & 0 & 2.67 & 10.67 & 0 & 0 & 0 \\ -1.67 & 0 & 0 & 0 & -1.67 & -6.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} + s \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right] \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_{1R} \\ x_{2R} \end{Bmatrix}$$

$$= \begin{bmatrix} -.2 & .5 & 0 & 1 & .16 & 0 \\ 0 & 0 & 0 & -.5 & 0 & 0 \\ 0 & .417 & 1 & .833 & .0333 & 0 \\ 0 & 2.5 & 0 & 5 & .8 & -1 \\ 0 & .833 & 0 & 1.67 & .267 & 0 \\ 0 & .667 & 0 & 1.33 & .133 & 0 \\ 0 & -.417 & 0 & -.833 & -.033 & 0 \\ 0 & 0 & 0 & 0 & .2 & 0 \\ 0 & -.417 & 0 & -.833 & -.0333 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(B - 13)}$$

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \end{Bmatrix} = \begin{bmatrix} -3.33 & 0 & 0 & 0 & -3.33 & -13.33 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1.67 & 0 & 0 & 0 & -2.67 & -10.67 & 0 & -4 & -1.6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.67 & 0 & 0 & 0 & 1.67 & 6.67 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_{1R} \\ x_{2R} \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_1 \\ q_2 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_5 \\ q_6 \end{Bmatrix} \quad \text{(B - 14}$$

Note that reduced rows and columns have been retained and packed into the lower rows and right hand columns of the matrix.

The general form is

$$\left[[A_0] + s\,[A_I]\right]\{y'\} = [V]\,\{U\}$$

$$\left[\left[\begin{array}{c|c} -[A] & [0] \\ n\times n & \\ \hline [0] & [I] \end{array}\right] + s\left[\begin{array}{c|c} [I] & [0] \\ n\times n & \\ \hline [0] & |A_I| \end{array}\right]\right] \begin{Bmatrix} x \\ \\ x_R \end{Bmatrix} = \underset{k\times m}{[V]}\,\{U\} = \underset{k\times m}{\left[\begin{array}{c} [B] \\ n\times m \\ \hline [B_R] \\ R\times m \end{array}\right]}\{U\}$$

$$\{y\} = \left[\begin{array}{c|c} [C] & [T_R] \\ k\times n & k\times R \end{array}\right] \begin{Bmatrix} x \\ \hline x_R \end{Bmatrix} \tag{B - 15}$$

where:

k original size of the problem (the apparent state variable size)

n size of the state variable matrix

R = n - k

m the number of input variables (the size of F)

The first n rows of the matrix yield,

$$[-A]\,\{x\} + s\,\{x\} = [B]\,\{U\}$$

or, (B - 16)

$$s\,\{x\} = [A]\,\{x\} + [B]\,\{U\}$$

If $[A_I]$ is zero, the last R rows yield,

$$\{x_R\} = [B_R]\,\{U\} \tag{B - 17}$$

The output equation is,

$$\{y\} = \left[C \,\vdots\, T_R\right] \begin{Bmatrix} x \\ x_R \end{Bmatrix} = [C]\ \{x\} + \left[T_R\right] \{x_R\}$$

$$= [C]\ \{x\} + \left[T_R\right] \left[B_R\right] \{U\}$$

$$\{y\} = [C]\ \{x\} + [D]\ \{U\} \qquad (B-18)$$

Summarizing,

- [A] is the negative of the first n rows and columns of $\left[A_0\right]$.
- [B] is the first n rows of [V].
- [C] is the first n columns of [T].
- [D] = $\left[T_R\right]$ $\left[B_R\right]$ where,
  $\left[T_R\right]$ - - last R columns of [T], and
  $\left[B_R\right]$ - - last R rows of [V].

For the sample problem, the results are as follows.

$$[A] = \begin{bmatrix} -2 & 0 & 0 & 0 & -.2 & -6.8 & 0 \\ 0 & -1 & 0 & 1 & 0 & 3 & 0 \\ -1.67 & 0 & -20 & 0 & -1.67 & 13.33 & -10 \\ -10 & 0 & 0 & -15 & -10 & -40 & 0 \\ -3.33 & 0 & 0 & 0 & -3.33 & -13.33 & 0 \\ -1.67 & 0 & 0 & 0 & -2.67 & -10.67 & 0 \\ 1.67 & 0 & 0 & 0 & 1.67 & 6.67 & 0 \end{bmatrix} \qquad (B-19)$$

$$[B] = \begin{bmatrix} -.2 & .5 & 0 & 1 & .16 & 0 \\ 0 & 0 & 0 & -.5 & 0 & 0 \\ 0 & .417 & 1 & .833 & .0333 & 0 \\ 0 & 2.5 & 0 & 5 & .8 & -1 \\ 0 & .833 & 0 & 1.67 & .267 & 0 \\ 0 & .667 & 0 & 1.33 & .133 & 0 \\ 0 & -.417 & 0 & -.833 & -.0333 & 0 \end{bmatrix} \qquad (B-20)$$

$$[C] = \begin{bmatrix} -3.33 & 0 & 0 & 0 & -3.33 & -13.33 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1.67 & 0 & 0 & 0 & -2.67 & -10.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0qq & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1.67 & 0 & 0 & 0 & 1.67 & 6.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad (B-21)$$

$$[D] = \begin{bmatrix} 0 & .833 & 0 & 1.67 & .267 & 0 \\ 0 & 0\ 0 & 0 & 0 & 0 & 0 \\ 0 & .677 & 0 & 1.33 & .133 & 0 \\ 0 & 0 & 0 & 0\ = & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.417 & 0 & -.833 & -.0333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (B-22)$$

$$\left[A_I\right] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (B-23)$$

$$\left[T_R\right] = \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ .4 & -1.6 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (B-24)$$

$$\left[B_R\right] = \begin{bmatrix} 0 & 0 & 0 & 0 & .2 & 0 \\ \\ 0 & -.417 & 0 & -.833 & -.0333 & 0 \end{bmatrix} \tag{B - 25}$$

The situation for $\left[A_I\right] \neq 0$ occurs when impulsive terms are presented in the original equations. These terms can not be accommondated in the usual state variable form. Consider the following matrix equation.

$$\begin{bmatrix} s & -s^2 \\ \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ \\ x_2 \end{Bmatrix} = \begin{Bmatrix} U_1 \\ \\ U_2 \end{Bmatrix} \tag{B - 26}$$

The transfer function can be obtained directly.

$$\begin{aligned} s\,x_1 - s^2\,x_2 &= U_1 \\ x_2 &= U_2 \\ s\,x_1 &= U_1 + s^2\,U_2 \\ x_1 &= (1/s)\,U_1 + s\,U_2 \end{aligned} \tag{B - 27}$$

If the matrix is put into first order form,

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ \dot{x}_2 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \\ \end{Bmatrix} \tag{B - 28}$$

and transformed to the state variable form,

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \\ \end{Bmatrix} \tag{B - 29}$$

$$\begin{Bmatrix} x_1 \\ \dot{x}_2 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} x_1' \\ x_2 \\ \dot{x}_2 \end{Bmatrix} \qquad \text{(B - 30)}$$

x_1' is the single state variable

$[A] = 0$

$[B] = \lfloor 1 \quad 0 \rfloor$

$$s \{x\} = [A] \{x\} + [B] \{U\} \;;\; s\,x = (0)x + \lfloor 1 \quad 0 \rfloor \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \;;\; sx = U_1 \qquad \text{(B - 31)}$$

$C = (1, 0, 0)$

Solving for the reduced variables,

$$\begin{bmatrix} 1 & 0 \\ -s & 1 \end{bmatrix} \begin{Bmatrix} x_2 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \qquad \text{(B - 32)}$$

$$\begin{Bmatrix} x_2 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & s \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \left[B_R'\right] \{U\} \qquad \text{(B - 33)}$$

$$[D] = \left[T_R\right]\left[B_R'\right] = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} 0 & s \\ 0 & s \\ 0 & 1 \end{bmatrix} \qquad \text{(B - 34)}$$

$$\{y\} = [C] \{x\} + [D] \{U\} \qquad \text{(B - 35)}$$

$$\begin{Bmatrix} x_1 \\ \dot{x}_2 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} x_1' + \begin{bmatrix} 0 & s \\ 0 & s \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \qquad \text{(B - 36)}$$

$$sx_1' = U_1 \; ; \; x_1' = U_1/s$$

$$x_1 = U_1/\left(s + sU_2\right) \; ; \; x_2 = sU_2 \; ; \; x_2 = U_2 \qquad \text{(B - 37)}$$

Therefore, if $\left[A_I\right] \neq 0$, the matrix $\left[s \left[A_I\right] + [I]\right]$ must be inverted and used to premultiply $\left[B_R\right]$.

$$\left[B_R'\right] = \left[s \left[A_I\right] + [I]\right]^{-1} \left[B_R\right] \qquad \text{(B - 38)}$$

This special case is usually not encountered in practice and no provision is made for forming $\left[B_R'\right]$ in the computer program. The [D] matrix is computed as if $\left[A_I\right]$ were zero. However, the $\left[A_I\right]$, $\left[T_R\right]$ and $\left[B_R\right]$ matrices are preserved. For special applications, the user could compute $\left[B_R'\right]$.

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16. Abstract

This document describes the computer program known as QR. Classical control systems analysis and synthesis (root locus, time response, and frequency response) can be performed using this program. Volume I discusses program usage and includes a limited theoretical discussion. Volume II describes programming details of the QR computer program.

17. Key Words

QR
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Phase Locus
Frequency Response
Time Response
Power Spectral Density (PSD)

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